Calculation of power output and total efficiency for a closed gas turbine cycle, one axis.

In this cycle heat is transferred to air in a closed cycle by heating from a coal-fired boiler. The cycle has two compressors with inter-cooling, regenerator and after-cooler (before air entering the low pressure compressor). Start with drawing a connection scheme for the cycle.

Compressor pressure ratio (π) (both) 2.5
Inlet temperature each compressor 25ºC
Inlet temperature turbine 650ºC
Lowest pressure in cycle 8 bar
Isentropic efficiency compressor (both) \( \eta_{sk} = 0.85 \)
Isentropic efficiency turbine \( \eta_{st} = 0.90 \)
Pressure loss boiler (air heating) 2 %
Pressure loss, intercooler 5 %
Pressure loss, regenerator (both sides) 5 %
Pressure loss, after cooler 5 %
Effectiveness of regenerator* 80 %
Generator efficiency \( \eta_g = 0.98 \)
Mechanical efficiency \( \eta_m = 0.98 \)
Mass flow in the cycle 185 kg/s

The combustion efficiency is 82% and is defined as the ratio of utilized heat in the boiler to fuel supply

*Calculate the power output total electrical efficiency.

*This is the same as temperature efficiency for heat exchangers

Appendix 1: Specific heat ratio as function of gas content and temperature for flue gases

Nomenclature:

\( \kappa \) = ratio of specific heat \((C_p/C_V)\)
\( \pi \) = pressure ratio
\( x \) = gas content in flue gases
\( t \) = temperature in ºC
\( T \) = temperature in K.

Subscripts:

s = isentropic
K = compressor
T = turbine
REG = regenerator
m = mean value (average)
Appendix 1: Specific heat ratio as function of gas content and temperature for flue gases

![Graph showing specific heat ratio as a function of gas content and temperature.](image)

1 “Tabeller and diagram” page 171, version of year 1998 by L. Wester, Sweden, or any other arbitrary handbook with thermal engineering formulas and tables.
HINTS:

**Hint 1 (-3p):** The layout of the gas turbine is shown in the figure.

1-2: Compression no 1
2-3: Inter cooling
3-4: Compression no 2
4-5: Air heater from GT exhaust
5-6: Heat exchange with coal-fired boiler
6-7: Expansion in turbine
7-8: Cooling of GT exhaust air- Regeneration to compressed air
8-1: After cooling of GT exhaust before entering the compressor

**Hint 2 (-3p):** The power output is calculated from

\[ P_{el} = (P_T \cdot \eta_m - P_K) \cdot \eta_g \]

where the power output/input is the massflow*the specific heat for air*temperature difference.

**Hint 3 (-4p):** The temperature increase in the compressor can be calculated as

\[ \Delta T_K = T_2 - T_1 = \frac{T_1}{\eta_{sk}} \left[ \frac{(\kappa - 1)}{\kappa} \right] \]

Remember that \( \kappa \) is a function of the temperature (appendix 1), thus you have to iterate between \( \kappa \) and \( \Delta T \). Start assuming that \( \kappa \) is 1.40 for the first compressor.

The same procedure has to be done for the turbine when calculating the temperature decrease.
Solution

5) The figure shows the layout of the closed gas turbine

1-2: Compression no 1
2-3: Inter cooling
3-4: Compression no 2
4-5: Air heater from GT exhaust
5-6: Heat exchange with coal-fired boiler
6-7: Expansion in turbine
7-8: Cooling of GT exhaust air- Regeneration to compressed air
8-1: After cooling of GT exhaust before entering the compressor

The cycle in a T-s diagram is presented below:
The power output from the turbine can be calculated as the turbine power minus the compressors’ power:

\[ P_{el} = (P_T \cdot \eta_m - P_K) \cdot \eta_g \]

Power = mass flow * enthalpy difference

Enthalpy difference = \( c_p (\text{mean value for temperature interval}) \cdot \text{temperature difference} \)

\[ P_{el} = (m \cdot c_p \cdot \Delta T_T \cdot \eta_m - 2 \cdot m \cdot c_p \cdot \Delta T_K) \cdot \eta_g \]

\( c_p, \Delta T_T, c_p, \Delta T_K \)?

Temperature increase in the compressor \( \Delta T_K \)

\[ \Delta T_K = T_2 - T_1 = \frac{T_1}{\eta_{SK}} \left[ \frac{\kappa^{\frac{\kappa-1}{\kappa}}}{\kappa} \right] \]

\( T_1 = 25 + 273 = 298 K \)

\( \eta_{SK} = \text{compressor isentropic efficiency} = 0.85 \)

\( \pi_K = \text{compressor pressure ratio} = 2.5 \)

\( \kappa = \text{specific heat ratio} \)

In this equation we have two unknowns; we do not know \( \kappa \) or the temperature \( T_1 \). However \( \kappa \) is a function of the temperature increase, so we have to start an iteration process (appendix 1)

Assume \( \kappa=1.4 \) (\( x = 0 \), air at low temp), then calculate \( \Delta T_K \) and read the new \( \kappa \) value, iterating till the temperatures difference is no bigger than 5

\[ \Delta T_K = \frac{298}{0.815} \left[ 2.5 \left( \frac{0.4}{77.5^2} \right) - 1 \right] = 105 K \]

\( t_m = 25 + \frac{105}{2} = 77.5^\circ C \)

With \( t_m = 77.5^\circ C \) and \( x = 0 \) we find a new \( \kappa \).

\( \kappa = 1.397 \Rightarrow \Delta T_K = 104 K \)

OK! Then \( t_2 = 25 + 104 = 129^\circ C \)

We can find the specific heat \( C_{pk} \) for air with an average temperature of 77.5°C as²

² For example “Tabeller and diagram” page 167, version of year 1998 by L. Wester, Sweden, or any other arbitrary handbook with thermal engineering formulas and tables.
\[ c_{p_k} = 1008 \text{ J/kg, } K \approx 1.01 \text{kJ/kg, K} \]

**Temperature increase in turbine \( \Delta T_T \):**

Here we have to take into regards the pressure losses in the cycle, i.e. the turbine pressure ratio is not equal to the compressor ratio, as we loose pressure in the heat exchangers.

\[
\Delta T_T = T_6 - T_7 = \eta_{ST} \times T_6 \left[ 1 - \frac{1}{\pi_T^{\kappa-1}} \right]
\]

\[ \eta_{ST} = \text{turbine isentropic efficiency} = 0.90 \]

\[ \pi_T = \text{turbine pressure ratio} = \frac{P_6}{P_7} \]

\[
= \frac{p_i \times \pi_K \times (1 - \xi_{ic}) \times \pi_K \times (1 - \xi_{REG}) \times (1 - \xi_{boiler})}{p_i \times \frac{1}{(1 - 3_{AC})} \times \frac{1}{(1 - 3_{REG})}} \\
= \frac{2.5 \times 0.95 \times 2.5 \times 0.95 \times 0.98}{0.95 \times 0.95} = 4.99
\]

\[ 3 = \text{pressure loss coefficient} \]

\[ T_6 = 650 + 273 = 923K \]

Again we have one equation and 2 unknowns, so we iterate \( \kappa \) for expanding flue gases (in turbine) and temperature decrease, starting with assuming

\[ \kappa = 1.35 \]

(remember that the gas content \( x=0 \) in this configuration, we are working with a closed air cycle)

\[ \Delta T_T = 0.90 \times 923 \left[ 1 - \frac{1}{4.99^{0.95/1.35}} \right] = 283K \]

\[ t_m = 650 - \frac{283}{2} = 508^\circ C \]

\[ \Rightarrow \kappa = [x = 0.508^\circ C] = 1.356 \Rightarrow \Delta T_T = 286K \]

\[ t_m = 507^\circ C \]

OK!

The Specific heat \( C_{pt} \) for the expanding flue gases: 
\( (x = 0.507^\circ C) \)
\( c_p = 1093 \text{ J/kg, } K \approx 1.09 \text{ J/kg, } K \)

**Electrical Power:**

\[ m = \text{mass flow} = 185 \text{ kg/s} \]

\[ P_{el} = (185 \times 1.09 \times 286 \times 0.98 - 2 \times 185 \times 1.01 \times 104) \times 0.98 \]

\[ = 17300kW = 17.3MW \]

**Total efficiency:**

\[ \eta_{tot} = \frac{P_{useful}}{P_{fuel}} = \frac{P_{el}}{m \cdot c_p (t_6 - t_5) \cdot \eta_{comb}} \]

\[ \eta_{comb} = 0.82 \]

\[ t_6 = 650^\circ \]

\[ t_5 = ? \]

**Heat exchanger efficiency** \( \eta_{REG} = 0.80 \)

\[ \eta_{REG} = \frac{t_5 - t_4}{t_7 - t_4} = \text{heated air/ inlet temp. difference} \]

\[ \eta_2 = \frac{\Delta_2}{\theta} \]

\[ t_4 = t_3 + \Delta T_K = 25 + 104 = 129^\circ C \]

\[ t_7 = t_6 - \Delta T_T = 650 - 286 = 364^\circ C \]

\[ t_5 = 0.80(364 - 129) + 129 = 317^\circ C \]

\[ C_p \left[ t_m = \frac{650 + 317}{2} = 484^\circ C \right] = 1084\text{ J/kg, } K \]

\[ \eta_{tot} = \frac{17300}{185 \times 1.08 \times (650 - 317) \times \frac{1}{0.82}} = 0.21 \]

Low efficiency due to high flue gas temperature and low turbine inlet temperature. The temperature is limited because of the air cooling of boiler tubes.