Axial Compressor Design Parameters

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Course MJ2429

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Denotation</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>Absolute velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$h$</td>
<td>Enthalpy</td>
<td>J/kg</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass flow rate</td>
<td>kg/s</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>$u$</td>
<td>Tangential velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$w$</td>
<td>Relative velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Absolute flow angle</td>
<td>deg</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Relative flow angle</td>
<td>deg</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rotational speed</td>
<td>rad/s</td>
</tr>
<tr>
<td>$C$</td>
<td>Normalized absolute velocity</td>
<td>-</td>
</tr>
<tr>
<td>$W$</td>
<td>Normalized relative velocity</td>
<td>-</td>
</tr>
<tr>
<td>$U$</td>
<td>Normalized tangential velocity</td>
<td>-</td>
</tr>
</tbody>
</table>

Subscripts

0  Total
1  Inlet rotor
2  Outlet rotor (inlet stator)
3  Outlet stator
n  Normal
r  Radial component
x  Axial component
$\theta$  Tangential component
Compressor Stage Denotations and Conventions

Stage denotations

1 rotor inlet
2 stator inlet
3 stator outlet

Stage velocity triangles

Velocity triangles denotations and conventions

Relative flow angle

Axial velocity component (absolute and relative)

Relative velocity

Axial direction

Relative circumferential velocity component

Absolute circumferential velocity component

Circumferential speed
Stage Velocity Triangles

The stage velocity triangles are commonly employed as graphical measure to represent averaged kinetics of the flow at a reference radial position throughout the stage.

Usually one of the following reference radial positions is used:

- Mean radius

\[ r_m = \frac{r_h + r_s}{2} \]  

Eq. 1

- Euler radius (radius that splits the annular cross section in half)

\[ r_E = \sqrt{\frac{r_h^2 + r_s^2}{2}} \]  

Eq. 2

The absolute frame of reference is bound to the stator and is therefore non-rotating. The relative frame of reference is bound to the rotor and rotates with the circumferential speed of the rotor \( u \) at the reference radius obtained from

\[ u = r_{ref} \cdot \omega \]  

Eq. 3

The relation between the velocities in the absolute frame of reference (denoted “absolute velocities”) and the ones in the relative frame of reference (respectively denoted “relative velocities”) is the following

\[ w_x = c_x \]  

Eq. 4

\[ w_\theta = c_\theta - u \]  

Eq. 5

where \( c_x \) and \( c_\theta \) are the axial and circumferential components of the respective velocity as follows

\[ c^2 = c_x^2 + c_\theta^2 \]  

Eq. 6

\[ w^2 = w_x^2 + w_\theta^2 \]  

Eq. 7

The flow angles are defined as

\[ \tan \alpha = \frac{c_\theta}{c_x} \]  

Eq. 8

\[ \tan \beta = \frac{w_\theta}{w_x} \]  

Eq. 9

Note:

- The relative velocity is the velocity that an observer sees while sitting on the rotor
- The rotor blades thus see the relative flow velocities
- The direction of the absolute flow velocity at stator outlet corresponds approximately to the stator blade metal angle at the trailing edge
- The direction of the relative flow velocity at rotor outlet corresponds approximately to the rotor blade metal angle at the trailing edge
First Design Parameter: Degree of Reaction

The degree of reaction relates the change in enthalpy effectuated in the rotor to the change in enthalpy of the stage as follows

\[ R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}} \quad \text{Eq. 10} \]

This can be rewritten as

\[ R = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}} + \Delta h_{\text{stator}}} = \frac{h_2 - h_1}{h_2 - h_1 + h_3 - h_2} \quad \text{Eq. 11} \]

The change in enthalpies in stator and rotor respectively are related to the velocities as follows

In the rotor the rothalpy \( I = h + \frac{w^2}{2} - \frac{u^2}{2} \) is constant, thus

\[ h_2 + \frac{w_2^2}{2} - \frac{u_2^2}{2} = h_1 + \frac{w_1^2}{2} - \frac{u_1^2}{2} \quad \text{Eq. 12} \]

leading to

\[ h_2 - h_1 = \frac{1}{2} \left( \frac{w_1^2 - w_2^2 - u_1^2 + u_2^2}{2} \right) \quad \text{Eq. 13} \]

Note

- \( \frac{1}{2} \left( \frac{w_1^2 - w_2^2}{2} \right) \) is the contribution arising from the deceleration of the relative flow
- \( \frac{1}{2} \left( \frac{u_2^2 - u_1^2}{2} \right) \) is the contribution arising from the centrifugal effect

In the stator the stagnation enthalpy \( h_0 = h + \frac{c^2}{2} \) is constant, thus

\[ h_3 + \frac{c_3^2}{2} = h_2 + \frac{c_2^2}{2} \quad \text{Eq. 14} \]

leading to

\[ h_3 - h_2 = \frac{1}{2} \left( \frac{c_2^2 - c_3^2}{2} \right) \quad \text{Eq. 15} \]

Substituting these expressions into the equation of stage reaction above leads to the following general expression

\[ R = \frac{\frac{w_1^2 - w_2^2 - u_1^2 + u_2^2}{w_1^2 - w_2^2 - u_1^2 + u_2^2 + c_2^2 - c_3^2}} \quad \text{Eq. 16} \]
For a normal repetition stage with the following restrictions

\[ \bar{c}_1 = \bar{c}_3 \]  
\[ c_{x,1} = c_{x,2} = c_{x,3} = \text{const} \]  
\[ u_2 = u_3 \]

Eq. 17  
Eq. 18  
Eq. 19

the expression of the degree of reaction can further be simplified.

Firstly it can be noted that the circumferential speed \( u \) cancels out. Secondly the velocities shall be written in terms of their components as \( c^2 = c_x^2 + c_\theta^2 \) and \( w^2 = c_x^2 + w_\theta^2 \) respectively. This yields the following expression

\[
R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{w_{\theta,2}^2 - w_{\theta,2}^2 + c_\theta^2 - c_\theta^2} 
\]

Eq. 20

The relative velocity components in the denominator shall be expressed by the absolute velocity components as \( w_\theta = c_\theta - u \) leading to

\[
R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{c_\theta^2 - 2c_\theta u + u^2 - c_\theta^2 + 2c_\theta^2 u - u^2 + c_\theta^2 - c_\theta^2} 
\]

Eq. 21

After canceling out elements the expression can be rewritten as

\[
R = \frac{w_{\theta,1}^2 - w_{\theta,2}^2}{2u(c_\theta - c_\theta)} 
\]

Eq. 22

At this stage the enumerator shall be expressed as \( w_{\theta,3}^2 - w_{\theta,2}^2 = (w_{\theta,3} - w_{\theta,2})(w_{\theta,3} + w_{\theta,2}) \). The absolute velocity components in the denominator shall be expressed in terms of relative velocities as \( c_\theta = w_\theta + u \) leading to

\[
R = \frac{(w_{\theta,1} - w_{\theta,2})(w_{\theta,1} + w_{\theta,2})}{2u(w_{\theta,2} + u - w_{\theta,3} - u)} 
\]

Eq. 23

Both the circumferential speeds in the denominator and the relative components \( w_{\theta,1} - w_{\theta,2} \) cancel out finally yielding

\[
R = -\frac{1}{2u}(w_{\theta,1} + w_{\theta,2}) 
\]

Eq. 24
At this position a more intimate analysis of the degree of reaction is appropriate. For this purpose the relative circumferential velocity component at position 1 shall be expressed in the absolute frame of reference as \( w_{\theta} = c_{\theta} - u \) yielding

\[
R = -\frac{1}{2u} \left( c_{\theta,1} - u + w_{\theta,2} \right) = \frac{1}{2} \left( c_{\theta,1} + w_{\theta,2} \right) \tag{Eq. 25}
\]

By expressing the circumferential velocity components in terms of flow angles as \( \tan \alpha = \frac{c_{\theta}}{c_x} \) the following expression is obtained for the degree of reaction

\[
R = \frac{1}{2} \frac{c_x}{2u} \left( \tan \alpha_1 + \tan \beta_2 \right) \tag{Eq. 26}
\]

In the above equation the degree of reaction is expressed in terms of axial velocity component, circumferential speed and stator and rotor outflow angles respectively, which are approximately equal to blade metal angles at trailing edge (note that the assumption of repetition stage shall still be valid). According to the convention of velocity components depicted above flow angle \( \beta_2 \) is negative whilst flow angle \( \alpha_1 \) is positive. Usually the stage inflow angle \( \alpha_1 \) is close to zero, i.e. almost axial inflow. This leads to the following observations

- An increase in flow angle \( \beta_2 \) leads to an increase in degree of reaction \( \beta_2 \uparrow \Rightarrow R \uparrow \), i.e. the contribution of enthalpy change in the rotor to the total change in enthalpy in the stage gets larger
- An increase in flow angle \( \alpha_1 \) leads to an decrease in degree of reaction \( \alpha_1 \uparrow \Rightarrow R \downarrow \), i.e. the contribution of enthalpy change in the stator to the total change in enthalpy in the stage gets larger
- For compressor stages the degree reaction usually lies in the range \([0.5...1]\)
Second Design Parameter: Loading Factor

The loading factor relates the change in total enthalpy effectuated in the stage to the rotational speed as follows

\[ \psi = \frac{\Delta h_0}{u^2} \]  
\[ \text{Eq. 27} \]

Under application of Euler’s turbine equation the change in enthalpy can be expressed as \( \Delta h_0 = u_2 c_{\theta,2} - u_1 c_{\theta,1} \) leading to

\[ \psi = \frac{u_2 c_{\theta,2} - u_1 c_{\theta,1}}{u^2} \]  
\[ \text{Eq. 28} \]

For a normal repetition stage with the following restrictions

\[ \bar{c}_1 = \bar{c}_3 \]  
\[ c_{x,1} = c_{x,2} = c_{x,3} = \text{const} \]  
\[ u_2 = u_3 \]  
\[ \text{Eq. 29} \]
\[ \text{Eq. 30} \]
\[ \text{Eq. 31} \]

the expression of the loading factor can further be simplified to

\[ \psi = \frac{c_{\theta,2} - c_{\theta,1}}{u} \]  
\[ \text{Eq. 32} \]

Expressing the absolute flow velocities in the relative frame of reference as \( c_{\theta} = w_{\theta} + u \) the loading factor can be expressed as

\[ \psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} \]  
\[ \text{Eq. 33} \]

An equivalent expression can be obtained by substituting the relative velocity component at position 1 in the absolute frame of reference as \( c_{\theta} = w_{\theta} + u \) yielding

\[ \psi = 1 + \frac{w_{\theta,2} - w_{\theta,1}}{u} \]  
\[ \text{Eq. 34} \]

, which also can be expressed in terms of flow angles \( \beta_2 \) and \( \alpha_1 \) as follows

\[ \psi = 1 + \frac{c_x}{u} \left( \tan \beta_2 - \tan \alpha_1 \right) \]  
\[ \text{Eq. 35} \]

According to the convention of velocity components depicted above flow angle \( \beta_2 \) is negative whilst flow angle \( \alpha_1 \) is positive. This leads to the following observation:

- Decrease in flow angles \( \beta_2 \) and \( \alpha_1 \) lead to increase in loading factor (\( \beta_2, \alpha_1 \downarrow \Rightarrow \psi \uparrow \))
Third Design Parameter: Flow Coefficient

The flow coefficient relate the axial velocity component to the circumferential speed as follows

\[ \phi = \frac{c_x}{u} \]  

Eq. 36

The only observation to make for this coefficient is that the higher the axial velocity in the stage the higher the flow coefficient. As can be recognized below the flow coefficient stretches the velocity triangles in the axial direction.
The Normalized Velocity Triangle

At this position the normalized velocity triangle shall be introduced. The normalization consists therein that all velocity components are depicted with reference to the outlet circumferential velocity \( u_3 \). The normalized velocity components are denoted by the respective capital letters and yield from

\[
C = \frac{c}{u_3} \quad \text{Eq. 37}
\]

\[
W = \frac{w}{u_3} \quad \text{Eq. 38}
\]

\[
U = \frac{u}{u_3} \quad \text{Eq. 39}
\]

The special case of a normal repetition stage shall be regarded here for the sake of simplicity. The applied principle is however valid for all types of turbine stages.

Conveniently the velocity triangle is drawn with a common origin for stator and rotor outlet. As a normal repetition stage with the condition \( c_{x,1} = c_{x,2} = c_{x,3} = \text{const} \) is considered the height of the triangle corresponds to \( C_x = \frac{c_x}{u} = \phi \), i.e. the flow coefficient.

Note:

- The height of the velocity triangle corresponds to the flow coefficient \( \Phi \)
- The loading coefficient corresponds to the circumferential distance between \( C_2 \) and \( C_1 \). In the case of a repetition stage this equals to the circumferential distance between \( W_2 \) and \( W_1 \).
- The degree of reaction equals to the distance between axial and half the midpoint between \( W_2 \) and \( W_1 \).
Special Cases

The special cases are here analyzed for the case of normal repetition stage. Similar analysis can be performed in a general manner for other types of stages.

Degree of Reaction equal to one half (R=0.5)

The expression of the degree of reaction yields the following

\[ R = \frac{1}{2} = -\frac{1}{2u} (w_{\theta,1} + w_{\theta,2}) \Rightarrow w_{\theta,2} + u = -w_{\theta,1} \]

Eq. 40

which is equivalent to \( c_{\theta,2} = -w_{\theta,1} \). Substituting this expression into the equation of loading coefficient yields

\[ \psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} \Rightarrow \psi = \frac{2\cdot w_{\theta,2}}{u} + 1 = \frac{2c_{\theta,2}}{u} - 1 \]

Eq. 41

Velocity triangle

Note:

- As \( c_{\theta,2} = -w_{\theta,1} \) and normal stage it follows that \(|c_2| = |w_1|\) and with the assumption of repetition stage \( (c_1 = c_3) \) consequently \( \Delta h_{\text{rotor}} = \Delta h_{\text{stator}} \). The change in enthalpy in a reaction stage is thus equally split on rotor and stator.
- Both stator and rotor effectuate compression of the fluid and thus \( p_2 > p_1 \) and \( p_3 > p_2 \)
Zero Exit Swirl ($c_{\theta,3}=0$)

With $c_{\theta,3} = 0 = w_{\theta,3} + u \Rightarrow w_{\theta,3} = -u$ and $w_{\theta,3} = w_{\theta,1}$ the degree of reaction writes to

$$R = -\frac{1}{2u}(w_{\theta,1} + w_{\theta,2}) = \frac{1}{2} - \frac{w_{\theta,2}}{2u}$$  \hspace{1cm} \text{Eq. 42}

The loading coefficient yields from

$$\psi = \frac{w_{\theta,2} - w_{\theta,1}}{u} = \frac{w_{\theta,2}}{u} + 1$$  \hspace{1cm} \text{Eq. 43}

Combining the two expressions leads after reformulation to a relationship between degree of reaction and loading coefficient as follows

$$\psi = 2 \cdot (1 - R)$$  \hspace{1cm} \text{Eq. 44}

Velocity triangle

Note:

- The flow exists the stage purely axial, i.e. there is no swirl at stage exit
- For a zero exit swirl stage the degree of reaction and the loading factor are dependent
Simplified Off-Design Analysis

At this position a simplified off-design analysis shall be carried out. Ideally a compressor is operated at design point, i.e. the point at which the design has been carried out. At this point the flow angles are equal to the design flow angles (see velocity triangle) and the compressor operates at optimum performance. Off-design operation of a compressor might however occur under the following circumstances:

- Change in compressor rotational speed
- Change in flow rate

Note that the flow rate will establish as a function of the consumer after the compressor, i.e. the device that follows the compressor. In a gas turbine the consumer comprises combustion chamber and turbine. In case the compressor is supplying a network for pressurized air the consumer can be seen as the sum of a number of sub-consumers, e.g. pneumatic tools.

As has been shown above the loading coefficient can be represented by the following equation under the assumption of normal repetition stage

\[ \psi = 1 + \frac{c_x}{u} \left( \tan \beta_2 - \tan \alpha_1 \right) \]  

Eq. 45

In a rough approximation the flow angles \( \beta_2 \) and \( \alpha_1 \) can be assumed equal to the blade metal angles at trailing edge of rotor and stator respectively. As the compressor stage geometry does not change during operation the parameters in the parenthesis above can be replaced by constant leading to

\[ \psi = 1 + \frac{c_x}{u} k_1 \]  

Eq. 46

Employing the flow coefficient \( \phi = \frac{c_x}{u} \) this expression can further be simplified to

\[ \psi = 1 + \phi \cdot k_1 \]  

Eq. 47

When drawn in a flow rate – loading coefficient diagram the loci of possible compressor operation thus lie on a straight line with inclination \( k_1 \), as depicted below. Note that the same analysis is also valid for turbines.

\[
\begin{align*}
&k_1 > 0 \rightarrow \text{turbine} \\
&k_1 = 0 \rightarrow \text{neutral} \\
&k_1 < 0 \rightarrow \text{compressor}
\end{align*}
\]
The compressor operating line indicates that a higher flow rate leads to a lower loading factor and by this also to a lower increase in pressure. Note that this behavior is different for turbines as a higher flow rate also leads to a higher loading factor. To understand this easier turn the problem upside down: if you connect a turbine to a pressure plenum you will be able to increase the flow rate by increasing the pressure difference across the turbine.

A simplified off-design analysis can be obtained by normalizing the above equations by the values at design point such that the expression yields \( \frac{\psi}{\psi_D} = 1 \) and \( \frac{\phi}{\phi_D} = 1 \) at design point.

Normalizing the above expression by \( \psi_D \) leads to
\[
\frac{\psi}{\psi_D} = 1 + \frac{\phi \cdot k_1}{\psi_D} \quad \text{Eq. 48}
\]

The inclination \( k_1 \) is expressed by the design parameters at design point as is
\[
k_1 = \frac{\psi_D}{\phi_D}^{-1} \quad \text{Eq. 49}
\]

finally leading to
\[
\frac{\psi}{\psi_D} = 1 + \frac{\psi_D^{-1}}{\psi_D} \cdot \frac{\phi}{\phi_D} \quad \text{Eq. 50}
\]

Graphically the simplified off-design analysis yields the following operating characteristic:

Note:

- This is just a simplified off-design analysis and thus does not reflect the true conditions. The overall characteristic is however captured, i.e. lower pressure ratio at higher flow rate
- Towards low flow rates the operation of a compressor is limited by instability phenomena (rotating stall, surge)
- Towards high flow rates the operation is limited by choke, i.e. sonic conditions (\( M=1 \)) in narrowest cross section
Axial Compressor – Effect of Degree of Reaction

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Course MJ2429

Source:

Traupel, W., 1977
"Thermische Turbomaschinen"
Vol. 1, 3rd Ed.
ISBN 3-540-07939-4
Compressor Design Aspects

Damian Vogt
Course MJ2429

Axial Compressor Rotor Types

Rotors of drum and disk construction

Source [1]
Multiple Rotors

Abbreviations:
- L.P. low pressure
- H.P. high pressure

Advantages
- Single spool simple and robust construction
- Twin spool optimum speed for LP and HP compressor; LP usually rotates slower than HP

Source [1]

KTH/EKV/DV
Advantages of triple-spool engine:

- Optimum fan speed; fan speed is lower than LP and HO compressor (large diameter gives high blade tip speeds \(\rightarrow\) supersonic due to low speed of sound)
- Avoid having a geared fan (a gear box for these power ratings is heavy)
Different problem for power generation turbine:

- Power shaft must rotate at constant speed. At part load behavior the compressor would however work in a better operating point at part speed.
- Solution: free power turbine

Solar Turbines mechanical drive

Source [2]
Rotor Blade Mount

Comparison of compressor and turbine blade fixations:

- Turbine: fir tree
- Compressor: dove tail

Reason: higher load at higher temperatures for turbines, fir tree distributes loads more efficiently but is more costly
Stator Vane Mount

Variable Stage Geometry

Variable guide vanes: control of stage angle of all blades in a blade row

Purpose:
- Minimize incidence at off-design operation
- Pre-swirl for load regulation

Source [1]
Abb. 161. Kennfeld einer Wirbelmaschine mit negativem Vordrall bei Laufradneigungswinkeln \( \theta = -11^\circ \text{ bis } -5^\circ \) (nach C. Kreh).

Inlet guide vanes

Rotor

Stator


a drillfreie Zuströmung zum Laufrad, b negativer Vordrall (Gegenrad), c positiver Vordrall (Mitrad).

\( \omega \) Relativgeschwindigkeiten, \( c \) Absolutgeschwindigkeiten, \( u \) Umlaufgeschwindigkeiten, \( \theta \) betont sich auf die Vorwärtswendung, \( \phi \) Komponente in Umfangsrichtung, \( \alpha \) Komponente in Achsrichtung, \( \alpha_1, \alpha_2, \alpha_3 \) Anstellwinkel, \( \gamma_p \) Stofffließwinkel.

Source [3]
Surge Control (Bleed Valves)

Bleed valves are used to avoid compressor surge during fast acceleration. Their use during steady operation however would be very wasteful and is therefore not taken into consideration.
Boundary Layer Control


- Anstellwinkel $\phi = -5^\circ$ entspricht in Abb. 99: $\beta_1 = 25^\circ$, $\delta = 0^\circ$ (Rech.-Stell.).
- $\beta_2 = 30^\circ$ (Rech.-Stell.). $\delta = +35^\circ$ entspricht in Abb. 99: $\beta_2 = 65^\circ$.

Abb. 107. Typische Form der Druckverteilung auf der Kante eines Tragflügelprofils und Strömungsverlauf auf der Profiloberseite bei Ablösung.

Abb. 108. Druckverteilung und Grenzschichtausbildung an einem Spaltflügel.

Source [3]
Aspiration

(a) Without Suction  (b) With Suction

Source [4]
Blade Twist

Source [1]

Close to tip

$\mathbf{r} \uparrow \Rightarrow \mathbf{w_\theta} \uparrow$

Close to hub

$\mathbf{r} \downarrow \Rightarrow \mathbf{w_\theta} \downarrow$

Tip

Mid

Hub
References


