Radiation

Radiation

Thermal radiation.:
”That electromagnetic radiation emitted by a body as a result of its temperature”.

Thermal radiation is restricted to a limited range of the electromagnetic spectrum.

fig. 8.1 i Holman

Blackbody radiation

A *blackbody* is a perfect radiator.

Three characteristics:

- It absorbs all incident radiation
- It radiates more energy than any real surface at the same temperature
- The emitted radiation is independent of direction

Also:
Blackbody radiation obey certain simple laws

Stefan-Boltzmann's law

The total power radiated from a *blackbody* is calculated from *Stefan-Boltzmann´s law*:

$$E_b = \sigma \cdot T^4$$

where E_b = total power radiated per unit area from a *blackbody* (W/m^2)

$\sigma = 5.669 \cdot 10^{-8}$ $W/(m^2 \cdot K^4)$. (Stefan-Boltzmann constant)

T = absolute temperature (K)

Planck distribution law

The wavelength distribution of emitted blackbody radiation is determined from the *Planck distribution law*:

$$E_{\lambda,b}(\lambda,T)=\frac{C_1}{\lambda^5\cdot\left[\exp(C_2/(\lambda\cdot T))-1\right]}$$

where $C_1 = 2\pi \cdot h \cdot c_o^2 = 3.742 \cdot 10^8\ W \cdot \mu m^4 / m^2$

$C_2 = (h \cdot c_o / k) = 1.439 \cdot 10^4\ \mu m\ K$

λ = *wavelength* (μm)

T = *absolute temperature* (T)

h = Planck constant

k = Boltzmann constant
c_0 = speed of light in vacuum

Result of increasing temperature on radiation

- Higher intensity
- Shorter wavelength ⇔ higher frequency

Wien's displacement law

The wavelength of maximum emissive power is determined by *Wien's displacement law:*

$$\lambda_{max} \cdot T = C_3 = 2897.8 \; \mu m \; K$$

Radiation from real surfaces

Real surfaces:

- emit and absorb less than blackbodies
- reflect radiation
- emit and absorb differently depending on angle and wavelength
- do not obey the simple laws

Fig. 12.16 i Incropera

Spectral emissivity and total emissivity

To account for ”real surface”- behavior we introduce the *spectral emissivity*, defined by

$$E_\lambda(\lambda,T) = \varepsilon_\lambda(\lambda,T) \cdot E_{\lambda,black}(\lambda,T)$$

and the *total emissivity* defined by

$$E = \varepsilon \cdot E_b = \varepsilon \cdot \sigma \cdot T^4$$

(ε = integrated average)

Gray diffuse body

To simplify matters it is common to assume the emissivity to be independent on wavelength and direction. Such a surface is called a *gray diffuse body*, for which

$$E_\lambda(\lambda,T) = \varepsilon_\lambda \cdot E_{\lambda,black}(\lambda,T)$$

$$\varepsilon_\lambda = constant = \varepsilon \qquad (0 < \varepsilon < 1)$$

Absorptivity, reflectivity, transmittivity

Incident radiation may be *absorbed, reflected* or *transmitted*

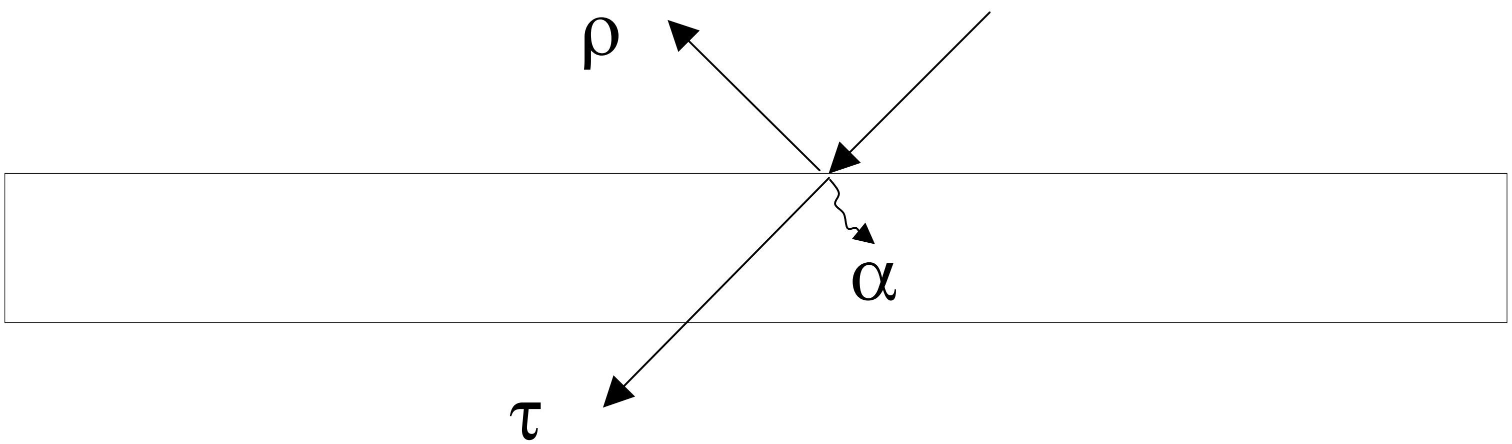
We define

Absorptivity α: Fraction of incident radiation absorbed

Reflectivity ρ: Fraction of incident radiation reflected

Transmittivity τ: Fraction of incident radiation transmitted

thus $$\alpha + \rho + \tau = 1$$



Kirchhoff's identity

The (total) emissivity and the (total) absorptivity of a surface are equal at equal temperatures:

$$\varepsilon = \alpha$$

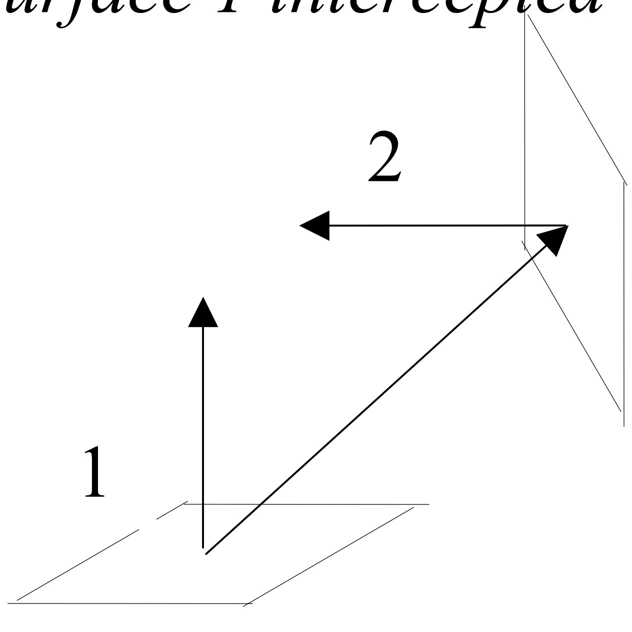
Radiation exchange between blackbodies

To calculate radiation exchange we must take into account

- surface areas
- surface geometries
- position in relation to each other

This is done by the *shape factor*, F_{12}

> F_{12} = *fraction of radiation leaving surface 1 intercepted by surface 2*.



Net exchange of radiation, law of reciprocity

Net exchange of radiation between blackbodies:

$$q_{1-2} = F_{12} \cdot A_1 \cdot (E_{b1} - E_{b2}) = F_{12} \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

Law of reciprocity

$$F_{12} \cdot A_1 \cdot = F_{21} \cdot A_2$$

Shape factors

Shape factors are often difficult to calculate
See diagrams and formulas in

Holman, Figs 8.12-8.16 and
CFT pp. 45-47

Important special case:
Small (convex) surface (1) surrounded by other surface (2):
⇨ $F_{12} = 1$.

Radiation exchange between real surfaces, simple case

For the special case of $F_{12} = 1$ the exchange is calculated as

$$q_{1\text{-}2} = \varepsilon_1 \cdot A_1 \cdot (E_{b1} - E_{b2}) = \varepsilon_1 \cdot A_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

Comparing to Newton's law of cooling:

$$q = h_r \cdot A_1 \cdot (T_1 - T_2)$$

⇨ $h_r = \varepsilon_1 \cdot \sigma \cdot (T_1^4 - T_2^4) / (T_1 - T_2) = \varepsilon_1 \cdot h_{r,\,black}$

⇨ $h_{r,\,black} = f(T_1,\ T_2)$, from table!

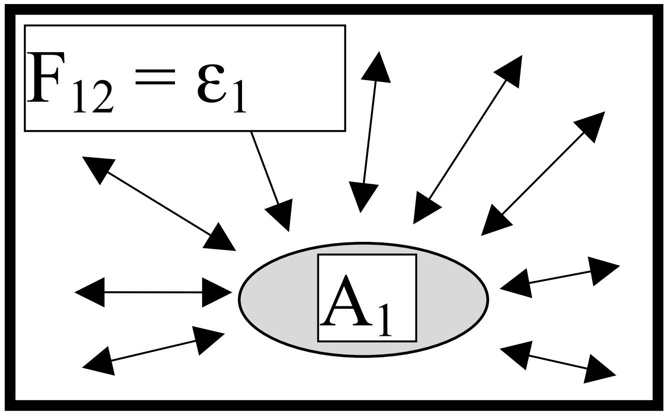


Table p.30 in CFT

Total emissivities of selected materials

Material	Temp (°C)	Emissivity ε
Aluminum, commercial sheet	100	0.09
Copper, polished	100	0.052
Iron, dark-gray surface	100	0.31
Glass, smooth	22	0.94
Snow-white enamel varnish	23	0.906
Black shiny lacquer	24	0.875
Roofing paper	21	0.91
Porcelain, glazed	22	0.92
Red brick	23	0.93
Al-paints	100	0.27-0.67

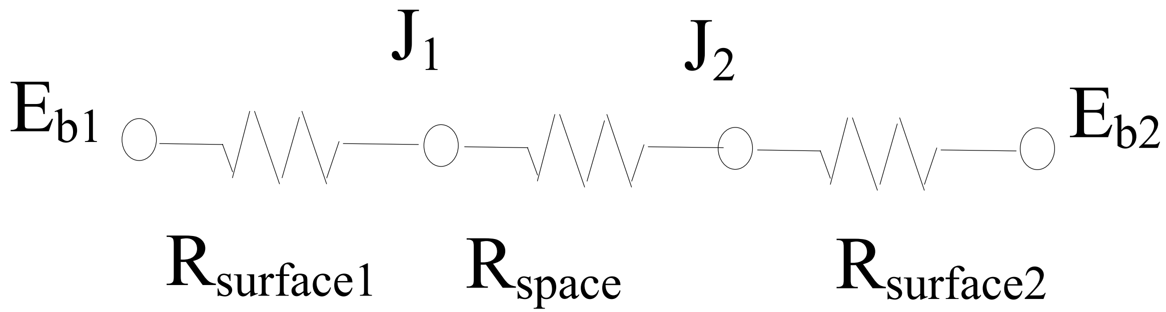
Radiation exchange calculated by resistance networks

Consider the blackbody emissive power as the driving potential

$$q_{1\text{-}2} = (E_{b1} - E_{b2}) / R_{rad} = \sigma \cdot (T_1^4 - T_2^4) / R_{rad}$$

Consider the radiation resistance as the sum of surface and space resistances

$$R_{rad} = R_{surface1} + R_{space} + R_{surface2}$$



Radiation exchange by resistance networks, assumptions

Assume:

- All surfaces are gray,
- All surfaces are uniform in temperature.
- Reflective and emissive properties are constant over the surfaces.

Define:

- *Irradiation,* G = total radiation / (unit time, unit area)
- *Radiosity*, J = total radiation leaving /(unit time, unit area) (including reflected radiation)

Assume these properties are uniform over each surface.

Surface resistance:

Radiosity = emitted radiation + radiation reflected:

$$J = \varepsilon \cdot E_b + \rho \cdot G$$

where ε = emissivity

ρ = reflectivity of surface

Assume surfaces opaque (τ=0)

=> $\rho = 1 - \alpha = 1 - \varepsilon$

(as $\alpha = \varepsilon$).

=> $J = \varepsilon \cdot E_b + (1 - \varepsilon) \cdot G$

or $G = (J - \varepsilon \cdot E_b) / (1 - \varepsilon)$

The net energy leaving the surface per unit area:

$$q / A = J - G = \varepsilon \cdot E_b + (1 - \varepsilon) \cdot G - G =$$
$$= \varepsilon \cdot E_b - \varepsilon \cdot G = \varepsilon \cdot (E_b - G)$$

=> $$q = \frac{\varepsilon \cdot A}{1 - \varepsilon} \cdot (E_b - J) = \frac{(E_b - J)}{(1 - \varepsilon) / (\varepsilon \cdot A)}$$

Consider (E_b - J) as the driving potential.

=> $R_{surface} = (1-\varepsilon)/(\varepsilon \cdot A)$

Space resistance:

Radiation from 1 to 2 (per unit time): $J_1 \cdot A_1 \cdot F_{12}$.

Radiation from 2 to 1 (per unit time): $J_2 \cdot A_2 \cdot F_{21}$.

Net exchange of radiation:

$$q_{12} = J_1 \cdot A_1 \cdot F_{12} - J_2 \cdot A_2 \cdot F_{21} = (J_1 - J_2) \cdot A_1 \cdot F_{12} = (J_1 - J_2) \cdot A_2 \cdot F_{21}$$

(as $A_1 \cdot F_{12} = A_2 \cdot F_{21}$).

Consider $(J_1 - J_2)$ as the driving potential,

=> $R_{space} = 1/(A_1 \cdot F_{12})$

Heat exchange due to radiation, two bodies

$$q = \frac{E_{b1} - E_{b2}}{R_{rad}} = \frac{E_{b1} - E_{b2}}{R_{surface1} + R_{space} + R_{surface2}} = \frac{\sigma \cdot (T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{\varepsilon_1 \cdot A_1} + \dfrac{1}{A_1 \cdot F_{12}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 \cdot A_2}}$$

Special case, two parallel infinite plates

$F_{12} = 1$ and $A_1 = A_2$

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Special case, two concentric cylinders or spheres

$F_{12} = 1$

$$\frac{q}{A_1} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \cdot (\frac{1}{\varepsilon_2} - 1)}$$

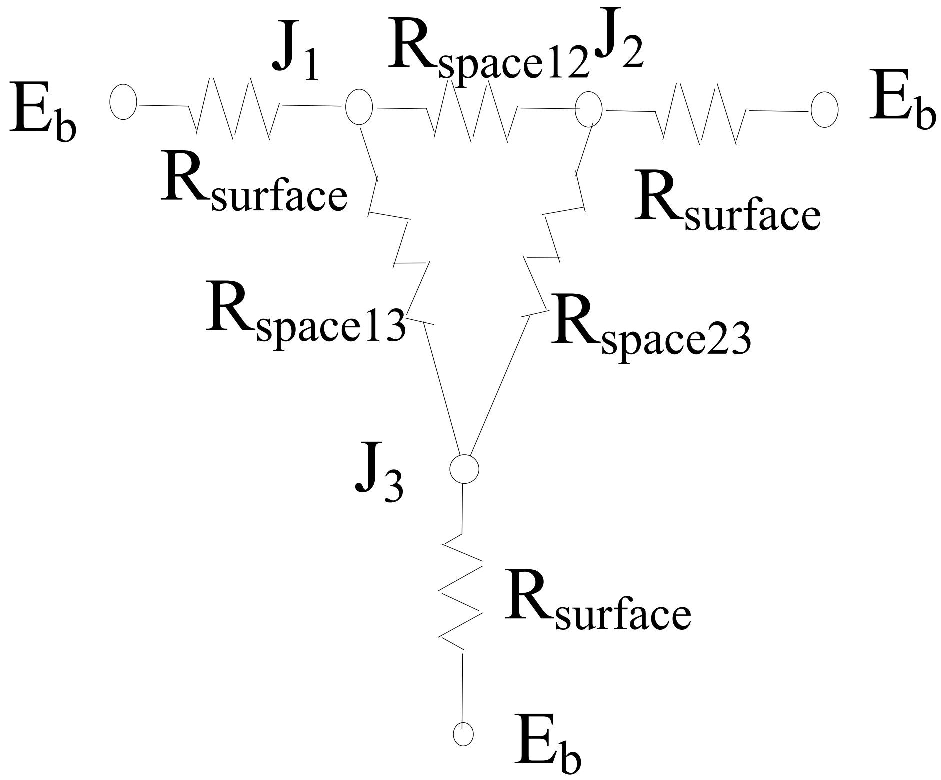
If $A_1 << A_2$ $\quad q / A_1 = \varepsilon_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$

Three body problem

- Calculate all resistances.
- For the nodes J_1, J_2 and J_3, the sum of the energy flows into each of the nodes must be zero.

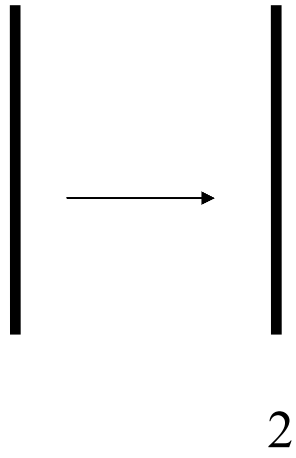
Example, node J_1

$$\frac{E_{b1}-J_1}{\left(\frac{1-\varepsilon_1}{\varepsilon_{1.}\cdot A_1}\right)}+\frac{J_2-J_1}{A_1\cdot F_{12}}+\frac{J_3-J_1}{A_1\cdot F_{13}}=0$$

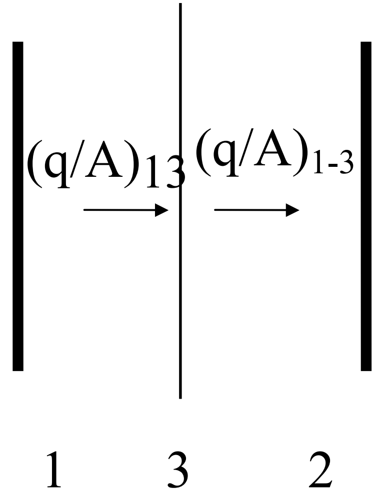


- Solve J_1 -J_3 !
- Calculate the heat exchanges!

Radiation shields



No shield



With shield

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

$$(q/A)_{1\text{-}3} = (q/A)_{3\text{-}2} = (q/A)$$

$$\frac{q}{A} = \frac{\sigma \cdot (T_1^4 - T_3^4)}{1/\varepsilon_1 + 1/\varepsilon_3 - 1} = \frac{\sigma \cdot (T_3^4 - T_2^4)}{1/\varepsilon_3 + 1/\varepsilon_2 - 1}$$

Everything is known except the temperature T_3.

Simplest case, all emissivities equal:

$$T_1^4 - T_3^4 = T_3^4 - T_2^4 \;\; => T_3^4 = ½ \cdot (T_1^4 + T_2^4)$$

$$=> \quad \frac{q}{A} = \frac{\tfrac{1}{2} \cdot \sigma \cdot (T_1^4 - T_2^4)}{1/\varepsilon_3 + 1/\varepsilon_2 - 1}$$

Emissivities assumed equal, => heat flux *reduced to half*

Multiple shields of equal emissivities

It may be shown by similar reasoning that the heat flux will be reduced to

$$(q / A)_{\text{with shields}} = \frac{1}{n+1} \cdot (q / A)_{\text{without shields}}$$

where n is the number of shields.

Different emissivities

Two plates with equal emissivities ε_1
One shield of different emissivity ε_2:

$$(q/A)_{\text{with shields}} = \frac{1}{2} \cdot \frac{\frac{2}{\varepsilon_1} - 1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} (q/A)_{\text{without shields}}$$

$\varepsilon_2 << \varepsilon_1$ => largest decrease in heat flux

Assume $\varepsilon_1 = 1$. =>

$$(q/A)_{\text{with shields}} = \frac{1}{2} \cdot \frac{2-1}{1 + \frac{1}{\varepsilon_2} - 1} (q/A)_{\text{without shields}} = \frac{1}{2} \cdot \varepsilon_2 \cdot (q/A)_{\text{without shields}}$$

thus equal to *one half the emissivity of the shield*.