**Exercise 1**: The microscopic cross-section for the capture of thermal neutrons by hydrogen is 0.33 b and for oxygen $2 \times 10^{-4}$ b. Calculate the macroscopic capture cross section of the water molecule for thermal neutrons assuming that water density is 1000 kg/m$^3$.

- Hint: use the formula for compounds:

$$\Sigma = \frac{10^3 \rho N_A}{M} (\nu_1 \sigma_1 + \nu_2 \sigma_2 + \ldots)$$

**Solution:**

```scilab
// Exercise 1 - solution with Scilab

// Find macroscopic capture cross section for water

sigCapHy = 0.33e-28; sigCapOx = 2.e-32;

// Number of H2O molecules per unit volume:

rhoH2O = 1000;
NA = 6.02e23; // Avogadro number
MolWeightH2O = 2 + 16; // Molecular weight of water

NH2OperUnitVolume = 1.e3*rhoH2O*NA/MolWeightH2O;

// Number of Oxygen nuclei = number of H2O molecules
NOperUnitVolume = NH2OperUnitVolume;

// Number of Hydrogen nuclei = 2 * number of H2O molecules
NHperUnitVolume = 2*NH2OperUnitVolume;

SIGCapHy = NHperUnitVolume*sigCapHy; // Macroscopic cross section
SIGCapOx = NOperUnitVolume*sigCapOx;

SIGH2O = SIGCapHy + SIGCapOx;

**Answer:** $SIGH2O = 2.208 \text{ m}^{-1}$
• **Exercise 2:** Disregarding the uranium-234, the natural uranium may be taken to be a homogeneous mixture of 99.28 weight percent of uranium-238 (absorption cross section 2.7 b) and 0.72 weight percent of uranium-235 (absorption cross section 681 b). The density of natural uranium metal is $19.0 \times 10^3$ kg m$^{-3}$. Determine the total macroscopic and microscopic absorption cross sections of this material.

   Hint: first find mass of U-235 and U-238 per unit volume of mixture and then number of nuclei per cubic meter of U-235 and U-238.

---

**Solution:**

```matlab
// Exercise 2 - solution with Scilab
//
// Finds the macroscopic and the microscopic cross section
// of the natural uranium
// barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn; sigAbsU235 = 681.0*barn;
//
// Density and weights
// rhoUran = 19000;
U238Weight = 0.9928;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number

// Mass of U235 and U238 per unit volume (density)
rhoU238 = rhoUran*U238Weight;
rhoU235 = rhoUran*U235Weight;

// Number of nuclei of U238 and U235 per unit volume
NU238 = 1.e3*rhoU238*NA/238;
NU235 = 1.e3*rhoU235*NA/235;

SIGUran = NU238*sigAbsU238 + NU235*sigAbsU235;
sigUran = SIGUran/(NU238+NU235);
```

**Answer:** Macroscopic cross section $\text{SIGUran} = 36.75$ m$^{-1}$, microscopic cross section $\text{sigUran} = 7.64$ barn.
**Exercise 3:** Calculate the thermal utilization factor for a homogenized core composed of (in % by volume): UO₂ 35% and H₂O 65%. The enrichment of the fuel is 3.2% (by weight). Microscopic cross sections [b] for absorption are as follows: water 0.66 [b], oxygen O: 2x10⁻⁴ [b], U-235: 681 [b], U-238: 2.7 [b].

Density of UO₂: 10200 kg/m³
Density of water: 800 kg/m³

**Solution:**

```scilab
// Exercise 3 – solution in Scilab
//
// Find the thermal utilization factor of a homogeneous reactor
// with 35% of UO2 and 65% of H2O

barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn;
sigAbsU235 = 681*barn;
sigAbsH2O  = 0.66*barn;
sigAbsOx   = 2.e-4*barn;

// Density and weights
rhoUO2 = 10200;
rhoH2O = 800;
U238Weight = 0.968;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number

// Partial Volumes of components
VolUO2 = 0.35;
VolH2O = 1. - VolUO2;

// Number of nuclei of U238 and U235 per unit volume of reactor
MUO2 = 235*U235Weight + 238*U238Weight + 32;
NUO2perReactVol = (1.e3*rhoUO2*NA/MUO2)*VolUO2;

// Number of Uranium nuclei (either U235 or U238) is the same as
// UO2 molecules, thus
NURanperReactVol = NUO2perReactVol;

// whereas O2 are twice as many
NOxperReactVol = 2*NUO2perReactVol;
```

Assumption: atoms of U235 and U238 are in the same proportion as enrichment. Calculate number of nuclei of U-235 and U-238 per unit volume of reactor

```
NU235perReactVol = NUranperReactVol*U235Weight;
NU238perReactVol = NUranperReactVol*U238Weight;
```

```
// Number of H2O molecules per unit volume of reactor
MH2O = 2 + 16;
NH2OperReactVol = (1.e3*rhoH2O*NA/MH2O)*VolH2O;
```

```
// Macroscopic absorption cross sections
SIGAbsU235 = NU235perReactVol*sigAbsU235;
SIGAbsU238 = NU238perReactVol*sigAbsU238;
SIGAbsH2O = NH2OperReactVol*sigAbsH2O;
SIGAbsOx = NOxperReactVol*sigAbsOx;
```

```
// Thermal utilization factor
f = SIGAbsU235/(SIGAbsU235+SIGAbsU238+SIGAbsH2O+SIGAbsOx);
```

**Answer:** thermal utilization factor $f = 0.843$

---

**Exercise 4:** Calculate the moderating power and the moderating ratio for H2O (density 1000 kg/m³) and Carbon (density 1600 kg/m³). The macroscopic cross sections are given below:

<table>
<thead>
<tr>
<th>Isotope</th>
<th>microscopic cross sections [b]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>absorption</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.332</td>
</tr>
<tr>
<td>Oxygen</td>
<td>27x10⁻⁵</td>
</tr>
<tr>
<td>Carbon</td>
<td>0.0034</td>
</tr>
</tbody>
</table>

**Solution:**

```
// Exercise 4 – solution in Scilab
```

```
// Calculate the moderating power and moderating ratio
// for water and graphite
// barn = 1.e-28; // define unit [b]
sigAbsH = 0.332*barn;
sigAbsC = 0.034*barn;
sigAbsOx = 2.7e-4*barn;
sigScatH = 38*barn;
```
sigScatC  = 4.75*barn;
sigScatOX = 3.76*barn;

// Define function for the Average Logarithmic Energy Decrement, ALED
function y = ALED(A), y = 1 + (A-1)^2*log((A-1)/(A+1))/2/A, endfunction

// Densities
rhoH2O  = 1000;
rhoC   = 1600;
NA    = 6.02e23; // Avogadro number

// Mass weights
MH2O = 2 + 16; MC = 12;

// Number of nuclei of H2O and C per unit volume
NH2O  = 1.e3*rhoH2O*NA/MH2O;
NC    = 1.e3*rhoC*NA/MC;

// Macroscopic cross sections
SIGAbsH2O  = NH2O*(2*sigAbsH + sigAbsOX);
SIGAbsC    = NC*sigAbsC;
SIGScatH2O = NH2O*(2*sigScatH + sigScatOx);
SIGScatC  = NC*sigScatC;

// average logarithmic energy decrement
ksiH = 1; ksiOX = ALED(16);
ksiC = ALED(12);
ksiH2O = (2*sigScatH*ksiH+sigScatOx*ksiOX)/(2*sigScatH+sigScatOx);

// Moderating powers
mpH2O = ksiH2O*SIGScatH2O;
mpC   = ksiC*SIGScatC;

// Moderating ratios
mrH2O = mpH2O/SIGAbsH2O;
mrC   = mpC/SIGAbsC;

**Answer:** Moderating power for H2O: mpH2O = 255.69; for Carbon: mpC = 6.02.
Moderating ratio for H2O: mrH2O = 115.09; for Carbon: mrC = 22.04.
• **Exercise 5:** Calculate the resonance escape probability for a reactor as in Exercise 3 assuming the fuel temperature $T = 1500$ K and the effective resonance integral for fuel at $T = 300$ K equal to 25 b. Microscopic cross sections for scattering are as follows: water 103 b, oxygen O: 6 b, U-235: 8 b, U-238: 8.3 b.

**Solution:**

```plaintext
// Exercise 5 – solution in Scilab

// Find the resonance escape factor of a homogeneous reactor
// with 35% of UO2 and 65% of H2O. Enrichment: 3.2%w

barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn; sigScatU238 = 8.3*barn;
sigAbsU235 = 681*barn; sigScatU235 = 8*barn;
sigAbsH2O = 0.66*barn;
sigAbsOx = 2.e-4*barn;
sigScatH = 38*barn;
sigScatOx = 3.76*barn;

// Calculate microscopic scattering cross section for H2O

sigScatH2O = 2*sigScatH + sigScatOx;

// Define function for the Average Logarithmic Energy Decrement, ALED
function y = ALED(A), y = 1 + (A-1)^2*log((A-1)/(A+1))/2/A, endfunction

// Density and weights

rhoUO2 = 10200;
rhoH2O = 800;
U238Weight = 0.968;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number

// Partial Volumes of components

VolUO2 = 0.35;
VolH2O = 1. - VolUO2;

// Number of nuclei of U238 and U235 per unit volume of reactor

MUO2 = 235*U235Weight + 238*U238Weight;
NUO2perReactVol = (1.e3*rhoUO2*NA/MUO2)*VolUO2;
```
Number of Uranium nuclei (either U\(^{235}\) or U\(^{238}\)) is the same as UO\(_2\) molecules, thus

\[
\text{NUuranperReactVol} = \text{NUO2perReactVol};
\]

whereas O\(_2\) are twice as many

\[
\text{NOxperReactVol} = 2\times\text{NUO2perReactVol};
\]

Assumption: atoms of U\(^{235}\) and U\(^{238}\) are in the same proportion as enrichment. Calculate number of nuclei of U-235 and U-238 per unit volume of reactor

\[
\begin{align*}
\text{NU235perReactVol} &= \text{NUuranperReactVol}\times\text{U235Weight}; \\
\text{NU238perReactVol} &= \text{NUuranperReactVol}\times\text{U238Weight};
\end{align*}
\]

Number of H\(_2\)O molecules per unit volume of reactor

\[
\text{MH2O} = 2 + 16;
\]

\[
\text{NH2OperReactVol} = (1.\times10^3\times\rho\text{H2O}\times\text{NA}/\text{MH2O})\times\text{VolH2O};
\]

Macroscopic absorption cross sections

\[
\begin{align*}
\text{SIGScatU235} &= \text{NU235perReactVol}\times\text{sigScatU235}; \\
\text{SIGScatU238} &= \text{NU238perReactVol}\times\text{sigScatU238}; \\
\text{SIGScatH2O} &= \text{NH2OperReactVol}\times\text{sigScatH2O}; \\
\text{SIGScatOx} &= \text{NOxperReactVol}\times\text{sigScatOx};
\end{align*}
\]

Total macroscopic cross section for scattering

\[
\text{SIGScatTot} = \text{SIGScatU235} + \text{SIGScatU238} + \text{SIGScatH2O} + \text{SIGScatOx};
\]

Microscopic cross sections for H\(_2\)O and Uranium

\[
\begin{align*}
\text{SIGScatUO2} &= \text{SIGScatU235} + \text{SIGScatU238} + \text{SIGScatOx}; \\
\text{sigScatUO2} &= \text{SIGScatUO2}/(\text{NU235perReactVol}+\text{NU238perReactVol}+\text{NOxperReactVol}); \\
\text{SIGScatU} &= \text{SIGScatU235} + \text{SIGScatU238}; \\
\text{sigScatU} &= \text{SIGScatU}/(\text{NU235perReactVol}+\text{NU238perReactVol});
\end{align*}
\]

Weighted average logarithmic decrement

\[
\begin{align*}
\text{ksiH} &= 1; \\
\text{ksiH2O} &= 1; \\
\text{ksiOx} &= \text{ALED}(16); \\
\text{ksiU235} &= \text{ALED}(235); \\
\text{ksiU238} &= \text{ALED}(238); \\
\text{ksiU} &= (\text{sigScatU235}\times\text{ksiU235}\times\text{U235Weight} + \text{sigScatU235}\times\text{ksiU235}\times\text{U235Weight})/(\text{sigScatU235}\times\text{U235Weight} + \text{sigScatU235}\times\text{ksiU235}); \\
\text{ksiUO2} &= (2\times\text{sigScatU}\times\text{ksiU}+2\times\text{sigScatOx}\times\text{ksiOx})/(\text{sigScatU}+2\times\text{sigScatOx});
\end{align*}
\]

Integral
\[
I = 25 \text{ barn} \times (1+6.0 \times 10^{-3} \times (\sqrt{1500} - \sqrt{300})) ;
\]
// Probability of the resonance escape
\[
p = \exp(-NUuranReactVol*I/ksi/SIGScatTot);
\]

Answer: the resonance escape probability \( p = 0.836 \)

---

Home Assignment #1

- Description and data for problems 1 and 2
  - A homogenized core has the following composition (in % by volume): UO\(_2\) - 32%, Zr - 10%, H\(_2\)O 58%. The enrichment of the fuel is 3.5% (weight). The material data are given in the Table below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Density (kg m(^{-3}))</th>
<th>( \sigma_v ) (barn)</th>
<th>( \sigma_f ) (barn)</th>
<th>( \sigma_t ) (barn)</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>H(_2)O</td>
<td>800</td>
<td>0.66</td>
<td>-</td>
<td>103</td>
<td>-</td>
</tr>
<tr>
<td>Zr</td>
<td>6500</td>
<td>0.18</td>
<td>-</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>UO(_2)</td>
<td>10200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U-235</td>
<td>18900</td>
<td>681</td>
<td>8</td>
<td>580</td>
<td>2.47</td>
</tr>
<tr>
<td>U-238</td>
<td>19000</td>
<td>2.7</td>
<td>-</td>
<td>8.3</td>
<td>-</td>
</tr>
<tr>
<td>O</td>
<td>-</td>
<td>2 \times 10^{-4}</td>
<td>-</td>
<td>6</td>
<td>-</td>
</tr>
</tbody>
</table>

Solution of HA#1:

// Home assignment #1 - solution with Scilab
//
// Find the thermal utilization factor of a homogeneous reactor
// with 32% of UO\(_2\), 10% of Zr and 58% of H\(_2\)O
// barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn; sigScatU238 = 8.3*barn;
sigAbsU235 = 681*barn; sigScatU235 = 8.*barn;
sigAbsH2O  = 0.66*barn; sigScatH2O  = 103*barn;
sigAbsOx   = 2.e-4*barn; sigScatOx   = 6*barn;
sigAbsZr   = 0.18*barn; sigScatZr   = 8*barn;
sigFisU235 = 580*barn;
niu = 2.47;
//
// Density and weights
//
rhoUO2 = 10200;
rhoH2O = 800;
rhoZr = 6500;
U238Weight = 0.965;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number
//
// Partial Volumes of components
//
VolUO2 = 0.32;
VolH2O = 0.58;
VolZr = 0.10;
//
// Number of nuclei of U238 and U235 per unit volume of reactor
// $MUO_2 = 235*U235\text{Weight} + 238*U238\text{Weight} + 32;$
NUO2perReactVol = (1.e3*rhoUO2*NA/MUO2)*VolUO2;
//
// Number of Uranium nuclei (either U235 or U238) is the same as
// UO2 molecules, thus
//
NUranperReactVol = NUO2perReactVol;
// whereas oxygen nuclei are twice as many
NOxperReactVol = 2*NUO2perReactVol;
//
// Assumption: atoms of U235 and U238 are in the same proportion
// as enrichment. Calculate number of nuclei of U-235 and U-238 per
// unit volume of reactor
//
NU235perReactVol = NUranperReactVol*U235Weight;
NU238perReactVol = NUranperReactVol*U238Weight;
//
// Number of H2O molecules per unit volume of reactor
//
MH2O = 2 + 16;
NH2OperReactVol = (1.e3*rhoH2O*NA/MH2O)*VolH2O;
//
// Number of Zr nuclei per unit volume of reactor
//
MZr = 91.2;
NZrperReactVol = (1.e3*rhoZr*NA/MZr)*VolZr;
//
// Macroscopic absorption cross sections
//
SIGAbsU235 = NU235perReactVol*sigAbsU235;
SIGAbsU238 = NU238perReactVol*sigAbsU238;
SIGAbsH2O  = NH2OperReactVol*sigAbsH2O;
SIGAbsOx   = NOxperReactVol*sigAbsOx;
SIGAbsZr   = NZrperReactVol*sigAbsZr;
//
// Problem 1: Thermal utilization factor
//
f = SIGAbsU235/(SIGAbsU235+SIGAbsU238+SIGAbsH2O+SIGAbsOx+SIGAbsZr);
write(%io(2),' Solution of Home Assignment #1')
write(%io(2),',---------------------')
write(%io(2),f,'(''f    = '',f10.4)')
//
// Problem 2: $k_{inf}$ - find the thermal utilization factor
//
//eta =
//NU235perReactVol*sigFisU235*niu/(NU235perReactVol*sigAbsU235+NU238perReactVol*sigAbsU238);
// SIGAbs for fuel must be the same as used in f factor; that is
SIGAbsU235
eta = NU235perReactVol*sigFisU235*niu/(NU235perReactVol*sigAbsU235);
kinf = f*eta*0.69*1.04;
write(%io(2),kinf,'(''kinf = '',f10.4)')
write(%io(2),',---------------------')
Answer: Output from the program

Solution of Home Assignment #1
--------------------------------
f = 0.8527
kinf = 1.2872
--------------------------------

- **Exercise 6:** Calculate the fuel temperature reactivity coefficient for a reactor as in Exercise 3 assuming the fuel temperature $T = 1500$ K and the effective resonance integral for fuel at $T = 300$ K equal to 25 [b]. Microscopic cross sections for scattering are as follows: hydrogen 38 [b], oxygen O: 3.76 [b], U-235: 8 [b], U-238: 8.3 [b].

Solution:
// Exercise 6 solved with Scilab
// Calculate the fuel reactivity coefficient of a homogeneous reactor
// with 35% of UO2 and 65% of H2O. Enrichment: 3.2%w
// barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn; sigScatU238 = 8.3*barn;
sigAbsU235 = 681*barn; sigScatU235 = 8*barn;
sigAbsH2O = 0.66*barn;
sigAbsOx = 2.e-4*barn;
sigScatH = 38*barn;
sigScatOx = 3.76*barn;
// Calculate microscopic scattering cross section for H2O
// sigScatH2O = 2*sigScatH + sigScatOx;
// Define function for the Average Logarithmic Energy Decrement, ALED
// function y = ALED(A), y = 1 + (A-1)^2*log((A-1)/(A+1))/2/A, endfunction
// Density and weights
// rhoUO2 = 10200;
rhoH2O = 800;
U238Weight = 0.968;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number

// Partial Volumes of components
VolUO2 = 0.35;
VolH2O = 1. - VolUO2;

// Number of nuclei of U238 and U235 per unit volume of reactor
MUO2 = 235*U235Weight + 238*U238Weight;
NUO2perReactVol = (1.e3*rhoUO2*NA/MUO2)*VolUO2;

// Number of Uranium nuclei (either U235 or U238) is the same as
// UO2 molecules, thus
NUranperReactVol = NUO2perReactVol;

// whereas O2 are twice as many
NOxperReactVol = 2*NUO2perReactVol;

// Assumption: atoms of U235 and U238 are in the same proportion
// as enrichment. Calculate number of nuclei of U-235 and U-238 per
// unit volume of reactor
NU235perReactVol = NUranperReactVol*U235Weight;
NU238perReactVol = NUranperReactVol*U238Weight;

// Number of H2O molecules per unit volume of reactor
MH2O = 2 + 16;
NH2OperReactVol = (1.e3*rhoH2O*NA/MH2O)*VolH2O;

// Macroscopic absorption cross sections
SIGScatU235 = NU235perReactVol*sigScatU235;
SIGScatU238 = NU238perReactVol*sigScatU238;
SIGScatH2O  = NH2OperReactVol*sigScatH2O;
SIGScatOx   = NOxperReactVol*sigScatOx;

// Total macroscopic cross section for scattering
SIGScatTot = SIGScatU235 + SIGScatU238 + SIGScatH2O + SIGScatOx;

// Microscopic cross sections for H2O and Uranium
SIGScatUO2 = SIGScatU235 + SIGScatU238 + SIGScatOx;
sigScatUO2 = SIGScatUO2/(NU235perReactVol+NU238perReactVol+NOxperReactVol);
SIGScatU = SIGScatU235 + SIGScatU238;
sigScatU = SIGScatU/(NU235perReactVol+NU238perReactVol);

// weighted average logarithmic decrement
ksiH = 1;
ksiOx = ALED(16); // 1 + (16-1)^2*log((16-1)/(16+1))/2/16;
ksiH2O = (2*sigScatH*ksiH+sigScatOx*ksiOx)/(2*sigScatH+sigScatOx);
ksiU235 = ALED(235);
ksiU238 = ALED(238);
ksiU = (sigScatU235*ksiU235*U235Weight+sigScatU235*ksiU235*U235Weight)/(sigScatU235*U235Weight+sigScatU235*ksiU235);
ksiUO2 = (2*sigScatU*ksiU+2*sigScatOx*ksiOx)/(sigScatU+2*sigScatOx);

// Integral
I = 25*barn*(1+6.e-3*(sqrt(1500)-sqrt(300)));
// Probability of resonance escape: NF number of U nuclei per system volume
NF = NUranperReactVol;
p = exp(-NF*I/ksi/SIGScatTot);
// Fuel Temperature reactivity coefficient:
alphaT = -NF*25*barn*1.6e-3/ksi/SIGScatTot/2/sqrt(1500);

**Answer:** the fuel temperature reactivity coefficient alphaT = -0.0000033 = -0.33 ppm
The differential equation that describes the iodine-135 concentration is as follows:

\[ \frac{dI}{dt} = \gamma \Sigma_f \phi - \lambda I \]

Since after reactor shut-down the neutron flux is zero, the equation becomes as:

\[ \frac{dI}{dt} = -\lambda I \]

So in summary, after shutdown the xenon-135 and iodine-125 concentrations are described with the following equations:

\[ \frac{dX}{dt} = \lambda X - \lambda I \]

Concentration of I-135 can be readily found as:

\[ I = I_0 e^{-\lambda t} \]

\[ \frac{dX}{dt} = \lambda I_0 e^{-\lambda t} - \lambda X \]

**Exercise 8:** Solve the differential equation that describes the xenon-135 concentration change after reactor shut-down

**Solution:** The differential equation that describes the xenon-135 concentration after reactor shut-down has been found in Exercise 7 and is as follows:

\[ \frac{dX}{dt} = \lambda I_0 e^{-\lambda t} - \lambda X \]
Multiplying both sides of this equation by so called integrating factor \(e^{\lambda t}\) yields

\[
\begin{align*}
\frac{dXe^{\lambda t}}{dt} &= \lambda_1 I_1 e^{\lambda t} \Delta e^{-\lambda_2 t} dt - \lambda_2 Xe^{\lambda t} dt \\
\frac{dXe^{\lambda t}}{dt} + \lambda_2 Xe^{\lambda t} dt &= \lambda_1 I_1 e^{\lambda t} \Delta e^{-\lambda_2 t} dt \\
\int \frac{dXe^{\lambda t}}{dt} &= \int \lambda_1 I_1 e^{\lambda t} \Delta e^{-\lambda_2 t} dt \\
Xe^{\lambda t} &= \frac{\lambda_1 I_1}{\lambda_2 - \lambda_3} e^{-\lambda_2 t} + C
\end{align*}
\]

here \(C\) is the integration constant, which is found using the condition: \(X = X_0 @ t = 0\).

After substitution, the constant is found as:

\[
C = X_0 + \frac{\lambda_1 I_0}{\lambda_2 - \lambda_3}
\]

and the expression for the xenon-135 concentration becomes:

\[
X = \frac{\lambda_1 I_1}{\lambda_2 - \lambda_3} e^{-\lambda_2 t} + \frac{\lambda_1 I_1}{\lambda_2 - \lambda_3} e^{-\lambda_3 t} + \frac{\lambda_1 I_0}{\lambda_2 - \lambda_3} e^{-\lambda_3 t} + X_0 e^{-\lambda_3 t}
\]

**Answer:** The xenon-135 concentration after reactor shut-down is described by the following equation:

\[
X(t) = \frac{\lambda_1 I_1}{\lambda_2 - \lambda_3} (e^{-\lambda_2 t} - e^{-\lambda_3 t}) + X_0 e^{-\lambda_3 t}
\]
• **Exercise 9:** Derive an expression for the time to attain the maximum concentration of xenon after shutdown

• Hint: At maximum, the time derivative of the concentration is equal to zero, that is: \( \frac{dX}{dt} = 0 \)

**Solution:**

• **Exercise 10:** A homogenized reactor as in Exercise 3 has been operating for a time at an average neutron flux of \( 2 \times 10^{18} \) neutrons/m²s; how long after shutdown will the xenon concentration reach a maximum and what is the concentration at that time? Given:

\[
\lambda_x = 2.9 \times 10^{-4} \text{ s}^{-1}, \quad \lambda_f = 2.1 \times 10^{-5} \text{ s}^{-1}, \quad \gamma_f = 0.061, \quad \gamma_x = 0.003, \quad \sigma_f = 580 \text{ [b]}, \quad \sigma_x = 2.6 \times 10^6 \text{ [b]}
\]

Hint: Use the number of atoms of U-235 per unit reactor volume found in exercise 3 to calculate the macroscopic fission cross section. Next use the expression derived in Exercise 9.

**Solution:**

**Home Assignment #2**

• A homogenized reactor as in Home Assignment #1 has been operating for a long time at an average neutron flux of \( 2 \times 10^{19} \) neutrons/m²s;

Given, in addition to HA1 data:

\[
\lambda_i = 2.9 \times 10^{-3} \text{ s}^{-1}, \quad \lambda_x = 2.1 \times 10^{-5} \text{ s}^{-1}, \quad \gamma_i = 0.061, \quad \gamma_x = 0.003, \quad \text{fission yield of iodine}, \quad \gamma_f = 0.003, \quad \text{fission yield of xenon,} \quad \sigma_f = 2.6 \times 10^6 \text{ [b]}, \quad \text{absorption cross section of xenon}
\]

**Problem 1 (5 points):**

– Plot the xenon-135 concentration as a function of time after reactor shut-down in a time range from 0 to 50 hours
– How long after shut-down will the xenon concentration reach a maximum and what is the concentration at that time?
Home Assignment #2

• Problem 2 (5 points):
  – For a poisoning effect on reactivity in a homogenized reactor one can use the following approximation:

  \[ \Delta \rho = -\frac{X}{\Sigma} = \frac{X\Sigma}{\rho} \]

  where \( X \) is the xenon concentration and \( \Sigma \) is the total absorption cross section in the reactor. What will be the maximum reactivity defect \( \Delta \rho \) after shut-down?

Solution of HA#2

// Home assignment #2 – solution with Scilab
//
// Calculate poison concentration after shut-down of a homogeneous reactor
// with 32% of UO2, 10% of Zr and 58% of H2O
//
// barn = 1.e-28; // define unit [b]
sigAbsU238 = 2.7*barn; sigScatU238 = 8.3*barn;
sigAbsU235 = 681*barn; sigScatU235 = 8.0*barn;
sigAbsH2O = 0.66*barn; sigScatH2O = 103*barn;
sigAbsOx = 2.e-4*barn; sigScatOx = 6*barn;
sigAbsZr = 0.18*barn; sigScatZr = 8*barn;
sigFisU235 = 580*barn;
niu = 2.47;
Nflux = 2.e19;
//
// Density and weights
//
rhoUO2 = 10200;
rhoH2O = 800;
rhoZr = 6500;
U238Weight = 0.965;
U235Weight = 1. - U238Weight;
NA = 6.02e23; // Avogadro number
//
// Partial Volumes of components
//
VolUO2 = 0.32;
VolH2O = 0.58;
VolZr = 0.10;
//
// Number of nuclei of U238 and U235 per unit volume of reactor
//
MUO2 = 235*U235Weight + 238*U238Weight + 32;
NUO2perReactVol = (1.e3*rhoUO2*NA/MUO2)*VolUO2;
//
// Number of Uranium nuclei (either U235 or U238) is the same as
// UO2 molecules, thus
//
NuranperReactVol = NUO2perReactVol;
whereas oxygen nuclei are twice as many

\[
\text{NOxperReactVol} = 2 \times \text{NUO2perReactVol};
\]

Assumption: atoms of U235 and U238 are in the same proportion as enrichment. Calculate number of nuclei of U-235 and U-238 per unit volume of reactor

\[
\text{NU235perReactVol} = \text{N UranperReactVol} \times \text{U235Weight};
\]
\[
\text{NU238perReactVol} = \text{N UranperReactVol} \times \text{U238Weight};
\]

Number of H2O molecules per unit volume of reactor

\[
\text{MH2O} = 2 + 16;
\]
\[
\text{NH2OperReactVol} = (1.e3 \times \text{rhoH2O} \times \text{NA} / \text{MH2O}) \times \text{VolH2O};
\]

Number of Zr nuclei per unit volume of reactor

\[
\text{MZr} = 91.2;
\]
\[
\text{NZrperReactVol} = (1.e3 \times \text{rhoZr} \times \text{NA} / \text{MZr}) \times \text{VolZr};
\]

Macroscopic absorption cross sections

\[
\text{SIGFisU235} = \text{NU235perReactVol} \times \text{sigFisU235};
\]
\[
\text{SIGAbsU235} = \text{NU235perReactVol} \times \text{sigAbsU235};
\]
\[
\text{SIGAbsU238} = \text{NU238perReactVol} \times \text{sigAbsU238};
\]
\[
\text{SIGAbsH2O} = \text{NH2OperReactVol} \times \text{sigAbsH2O};
\]
\[
\text{SIGAbsOx} = \text{NOxperReactVol} \times \text{sigAbsOx};
\]
\[
\text{SIGAbsZr} = \text{NZrperReactVol} \times \text{sigAbsZr};
\]
\[
\text{SIGAbs} = \text{SIGAbsH2O} + \text{SIGAbsZr} + \text{SIGAbsU235} + \text{SIGAbsU238} + \text{SIGAbsOx};
\]

Problem 1: Plot Xe-concentration as a function of time

\[
\text{lamI} = 2.9 \times 10^{-5}; \quad \text{lamX} = 2.1 \times 10^{-5}; \quad \text{gamI} = 0.061; \quad \text{gamX} = 0.003; \quad \text{sigAbsXe} = 2.6 \times 10^{6} \text{ barn};
\]
\[
\text{X0} = (\text{gamI} + \text{gamX}) \times \text{SIGFisU235} \times \text{Nflux} / (\text{lamX} + \text{sigAbsXe} \times \text{Nflux});
\]
\[
\text{I0} = \text{gamI} \times \text{SIGFisU235} \times \text{Nflux} / \text{lamI};
\]

function \( y = \text{XenConc}(t, \text{X0}, \text{I0}, \text{gamI}, \text{gamX}, \text{lamI}, \text{lamX}) \), \( y = \text{lamI} / (\text{lamX} - \text{lamI}) \times \text{I0} \times (\exp(-\text{lamI} \times t) - \exp(-\text{lamX} \times t)) + \text{X0} \times \exp(-\text{lamX} \times t) \), endfunction

\[
\text{T} = (0:0.5:50) \times 3600;
\]
\[
\text{XC} = \text{T};
\]

for \( i=1:length(\text{T}) \)
  \[
  \text{XC}(i) = \text{XenConc}(\text{T}(i));
  \]
end
plot(\text{T}/3600, \text{XC},'grid','on')
xlabel('Time, [h]')
ylabel('Xe-concentration, [atoms/m^3]')
write(%io(2), '' Solution of Home Assignment #2'')
write(%io(2), ''------------------------------------'')
write(%io(2), ''X0 = '',g10.4)''
write(%io(2), ''I0 = '',g10.4)''

Problem 2: Calculate the maximum reactivity defect due to poisoning

Time for maximum xenon concentration
//
TmaxXe = (1.-((lamX-lamI)/lamI*X0/I0)*log(lamX/lamI)/(lamX-lamI));
XeMax = XenConc(TmaxXe,X0,I0,gamI,gamX,lamI,lamX);
dRho = -XeMax*sigAbsXe/(SIGAbs+XeMax*sigAbsXe);
write(%io(2),dRho,'(''dRho = '',f10.4)'')
TmaxHr = TmaxXe/3600;
write(%io(2),TmaxHr,'(''Tmax = '',f10.4,'' hr'')')
write(%io(2),'--------------------------------')

**Answer:** output from the program

Solution of Home Assignment #2
--------------------------------
X0 = 0.3623E+22
I0 = 0.6217E+24
dRho = -0.7740
Tmax = 11.2254 hr
Exercise 11: In a cylindrical reactor with height $H = 3.6$ m and radius $R = 2$ m, a centrally-placed control rod is fully inserted into the reactor. Calculate the reactivity increase if the rod is withdrawn 15 cm from the reactor. The control rod worth $\Delta \rho(H)$ is equal to 2%.

Exercise 12: In a cylindrical reactor with height $H = 3.6$ m and radius $R = 2$ m, a control rod is placed at $r_0 = 1.2$ m from the centerline. The rod is half-way withdrawn. Calculate the reactivity increase if the rod is withdrawn with additional 15 cm from the reactor. The centrally-placed, identical control rod worth $\Delta \rho(H)$ is equal to 2%.

Solution:

Exercise 13: A cylindrical core has the extrapolated height and extrapolated radius equal to 3.8 m and 3.2 m, respectively. Calculate the volumetric heat source ratio between the point located in the middle of the central fuel assembly and the point located in a fuel assembly with a distance $r_f = 1.5$ m from the core center and at $z = 2.5$ m from the inlet of the assembly.

Solution:

Exercise 14: A cylindrical wall with the internal diameter equal to 8 mm and thickness 0.5 mm is heated on the internal surface with a heat flux equal to 1 MW m$^{-2}$. Calculate the temperature difference between the inner and the outer surface of the wall, assuming that the thermal conductivity of the wall material is equal to 16 W m$^{-1}$ K$^{-1}$. What will be the heat flux on the outer surface of the wall?
• **Exercise 15.** In a horizontal pipe with an internal diameter equal to 10 mm flows water. Calculate the wall shear stress change when mass flux of water increases from 50 to 2000 kg m\(^{-2}\) s\(^{-1}\). Assume that the water density and the dynamic viscosity are constant and equal to 740 kg m\(^{-3}\) and 1.0 \times 10^{-4} Pa s, respectively.

Solution:

• **Exercise 16.** For geometry as in the picture and given \(Q = 2.5 \times 10^4\) W, wall conductivity coefficient \(\lambda_1 = 16\) Wm\(^{-1}\)K\(^{-1}\), liquid temperature \(T_L = 323^\circ\)C, liquid mass flux \(G = 2000\) kg m\(^{-2}\) s\(^{-1}\).

1. Write shell balance for energy diffusion in the cylindrical solid,
2. state proper boundary conditions (boundary value problem),
3. calculate temperature distribution in the cylindrical solid,
4. Obtain the heat transfer coefficient from the Dittus-Boelter correlation. Conductivity, heat capacity and dynamic viscosity of water are constant values:
   \(\lambda_2 = 0.5\) Wm\(^{-1}\)K\(^{-1}\), \(\rho = 6 \times 10^3\) Jkg\(^{-1}\)K\(^{-1}\),
   \(\mu = 8.5 \times 10^{-5}\) kgm\(^{-1}\)s\(^{-1}\).

Solution:

• **Exercise 17.** For a channel as shown in the figure calculate the frictional pressure drop

Given:
- \(L = 1\) m, \(D_1 = 8\) mm, \(D_2 = 16\) mm, \(G = 103\) kgm\(^{-1}\)m\(^{-1}\)s\(^{-1}\), \(\rho = 660\) kgm\(^{-3}\), \(\mu = 8.53 \times 10^{-5}\) kgm\(^{-1}\)s\(^{-1}\),

To obtain the Fanning friction factor use the Blasius formula for smooth tubes

Solution:
Solution:

### Home Assignment #3 (1)

- **Problem 1 (5 points):**
  A pipe with an internal diameter 8 mm and wall thickness 1 mm is internally cooled with sub-cooled water and heated with an uniform heat flux 1MWm⁻² at the outer wall. Mass flux of the water is 1200 kg m⁻² s⁻¹, density 600 kg m⁻³, dynamic viscosity 6.8 10⁻⁵ Pa s, specific heat 8950 J Kg⁻¹ K⁻¹, heat conductivity 0.46 W m⁻¹ K⁻¹. Calculate the inner and the outer wall surface temperature in the pipe at the location where the bulk water temperature is equal 320 °C. Assume the thermal conductivity of wall 16 Wm⁻¹K⁻¹.

### Home Assignment #3 (2)

- **Problem 2 (5 points):**
  Calculate the total pressure drop in a vertical tube with total length 3.6 m (see Figure on the next page). The lower inlet part of the tube has the diameter equal to 55 mm. The upper part of the tube has the diameter equal to 57.5 mm. The sudden area change is located at 2.7 m from the inlet. The upward flowing fluid is water with the inlet mass flux $G = 1200$ kg m⁻² s⁻¹. Neglect inlet and outlet local losses. Neglect phase-change effects and assume constant fluid properties along the channel. Use the Blasius formula for the friction coefficient.

### Home Assignment #3 (3)

- **Problem 2 (cont’ed):**
  Water properties: density 740 kg m⁻³, dynamic viscosity $9.12 \times 10^{-5}$ Pa s

**Solution:**

```
// Home Assignment # 3 – solution with Scilab

Di = 0.008; Do = 0.01;
q2po = 1.e6;
G = 1200;
RHO = 600;
VISL = 6.8e-5;
CPL = 8950;
CONL = 0.46;
CONW = 16;
Tf = 320;

// Part 1: Find temperature at outer surface
q2pi = q2po*Do/Di; // Find heat flux at inner surface
Pr = CPL*VISL/CONL; // Find the Prandtl number
Re = G*Di/VISL; // Find the Reynolds number
Nu = 0.023*Re^0.8*Pr^0.33; // Find the Nusselt number
h = CONL*Nu/Do; // Heat transfer coefficient
```
Twi = Tf + q2pi/h; // Inner surface temperature
Two = Twi + q2po*Do*log(Do/Di)/2/CONW; // Outer surface temperature
write(%io(2),Two,'(''Outer surface temperature = '',f10.1,'' deg C'')')

// Part 2: Find total pressure drop
// Given data
G1 = 1200; L1 = 2.7; L2 = 0.9;
VISL = 9.12e-5; RHOL = 740;
D1 = 0.055; D2 = 0.0575;
pi = atan(1)*4; // Number PI = 3.1415....
A1 = pi*D1*D1/4; A2 = pi*D2*D2/4; // Cross-section areas
PW1 = pi*D1; PW2 = pi*D2; // Wetted perimeters
G2 = G1*A1/A2;
Rel = G1*D1/VISL; Re2 = G2*D2/VISL; // Reynolds numbers
Cf1 = 0.0792*Rel^(-0.25); Cf2 = 0.0791*Re2^(-0.25); // Fanning friction coefficients
Dpf1 = -Cf1*2*L1/D1*G1*G1/RHOL; // Friction loss in first pipe
Dpf2 = -Cf2*2*L2/D2*G2*G2/RHOL; // Friction loss in second pipe
DpextI = -(1-A1/A2)^2*G1*G1/2/RHOL; // Irreversible pressure loss in sudden expansion
DpextR = (G1*G1-G2*G2)/2/RHOL; // Reversible (Bernoulli) pressure drop in sudden expansion
Dpgrav = -RHOL*9.81*(L1+L2); // Gravitational pressure drop
Dptot = Dpf1+Dpf2+DpextI+DpextR+Dpgrav; // Total pressure drop
write(%io(2),Dptot,'(''Total pressure drop = '',f10.1,'' Pa'')')

Answer: Output from the program
Outer surface temperature = 455.1 deg C
Total pressure drop = -26641.0 Pa

• Exercise 18: A pipe with the inner diameter 10 mm and the outer diameter 12 mm is uniformly heated on the outer surface with heat flux q'' = 1 MW m^-2 and cooled with water flowing inside the pipe. The inlet mass flux of water is 1200 kg m^-2 s^-1, the inlet enthalpy 1215.6 kJ kg^-1 and the inlet pressure 70 bar. At what distance from the inlet the water will become saturated?

Solution:
• **Exercise 19:** In a pipe as in Exercise 18 assume that the single-phase flow exists from the inlet to the pipe until the point where water becomes saturated. Calculate the wall temperature on the outer surface of the pipe at the point where water becomes saturated if the wall conductivity is equal to 15 W m⁻¹ K⁻¹.

**Solution:**

• **Exercise 20:** For a fuel rod as in Example 7.3.1 (see Compendium, Chapter 7, page 7-8) assume that the fuel heat conduction is the following function of temperature, \( T \) [K],

\[
\dot{\lambda} = \frac{40}{130+T} + 0.34 \cdot 10^{-15} \cdot T^4 \quad [W \ m^{-1} \ K^{-1}]
\]

What will be the total temperature drop difference when the heat transfer coefficient drops from 5000 to 1300 W m⁻² K⁻¹?

**Solution:**
Exercise 21: Water flows in a horizontal pipe with a sudden expansion. Calculate the total pressure drop (reversible and irreversible) in a place where the pipe diameter suddenly increases from $A_1 = 10 \text{ cm}^2$ to $A_2 = 12 \text{ cm}^2$. Assume flow of water in direction from smaller to larger flow area with mass flow rate 1 kg/s and density 740 kg m$^{-3}$.

Solution:

Exercise 22: Solve the problem as in Exercise 21, but assume that the water flows in a direction from the larger to the smaller cross-section area (sudden contraction). Explain why the pressure drops in Exercises 21 and 22 are different.

Solution:

Exercise 24: Predict the location of the onset-of-nucleate-boiling point in a uniformly heated tube (8 mm internal diameter) with $q'' = 0.5 \text{ MW m}^{-2}$ on the internal wall. The tube is cooled with water at 140 bar, inlet subcooling 70 K and mass flux 2000 kg m$^{-2}$ s$^{-1}$. Use Bowring’s model for the onset of nucleate boiling and Jens and Lottes’ correlation for subcooled boiling. Use saturated water properties at 140 bar pressure. What will be the difference in the location of the ONB point if Thom et al. model is used instead?

Solution:

```scilab
// Exercise #24 – solution with Scilab
D = 0.008; pi = 4*atan(1); PH = pi*D; A = pi*D*D/4;
G = 2000;
```
Exercise 25: For conditions as described in Exercise 24, find the location of the point where the fully-developed boiling starts.

Solution:

//
// Solve Exercise #25
//
D = 0.008; pi = 4*atan(1); PH = pi*D; A = pi*D*D/4;
G = 2000;
p = 140;
dTsubi = 70;
q2p = 0.5e6;
//===================
cp = cplsat(p);
visc = vislsat(p);
con = conlsat(p);
//===================
Pr = cp*visc/con;
Re = G*D/visc;
//
// Solution
//
Nu = 0.023*Re^(0.8)*Pr^(0.33);
h = con*Nu/D;

dTsubONB1 = q2p/h - 25*(q2p/10^6)^0.25*exp(-p/62);  // Use Jens&Lottes
dTsubONB2 = q2p/h - 22.65*(q2p/10^6)^0.5*exp(-p/87);  // Use Thom et al.
zONB1 = cp*G*A*(dTsubi - dTsubONB1)/q2p/PH;
zONB2 = cp*G*A*(dTsubi - dTsubONB2)/q2p/PH;
write(%io(2),zONB1,' (zONB point with Jens&Lottes correlation ='' ,g10.4) ')
write(%io(2),zONB2,' (zONB point with Thom et al. correlation = '' ,g10.4) ')

Answer: output from the program
zONB point with Jens&Lottes correlation =  3.369 m
zONB point with Thom et al. correlation =  3.432 m
\[ dT_{\text{subFDB1}} = q'2p/h/1.4 - 25*(q'2p/10^6/1.4)^0.25*\exp(-p/62); \] // Use Jens&Lottes
\[ dT_{\text{subFDB2}} = q'2p/h/1.4 - 22.65*(q'2p/10^6/1.4)^0.5*\exp(-p/87); \] // Use Thom et al.

\[
\begin{align*}
z_{FDB1} &= cp^*G^*A^*(dT_{\text{subi}} - dT_{\text{subFDB1}})/q'2p/PH; \\
z_{FDB2} &= cp^*G^*A^*(dT_{\text{subi}} - dT_{\text{subFDB2}})/q'2p/PH;
\end{align*}
\]

Write(%io(2),zFDB1,'('''zFDB point with Jens&Lottes correlation = '',g10.4)''')
Write(%io(2),zFDB2,'('''zFDB point with Thom et al. correlation = '',g10.4)''')

**Answer:** output from the program
zFDB point with Jens&Lottes correlation = 3.684
zFDB point with Thom et al. correlation = 3.727

**Exercise 26:** Subcooled water at \( p = 70 \text{ bar} \) flows into a vertical round tube with uniformly heated wall with \( q'' = 200 \text{ kW m}^{-2} \) on the internal wall surface. The inlet subcooling is 40 K, the mass flux is 1200 kg m\(^{-2}\) s\(^{-1}\) and the tube internal diameter is \( D = 10 \text{ mm} \). Find the temperature distribution on the internal wall surface from the inlet to the point where the bulk water temperature is equal to the saturation temperature. Use saturated water properties at 70 bar pressure.

**Solution:**

**Exercise 27:** Plot \( q''(z) \) using the Bowring correlation and assuming steam-water flow at 70 bar in a pipe with \( D = 10 \text{ mm} \) and length 3.5 m. The inlet subcooling is 10 K, total mass flux \( G = 1250 \text{ kg m}^{-2} \text{ s}^{-1} \), and the correlation parameters are given as \( F_{B1} = F_{B2} = F_{B3} = F_{B4} = 1 \).

**Exercise 28:** Using the Levitan and Lantsman correlation for dryout (Eq. 8.3.5), plot \( x_{\text{crit}} \) as a function of pressure in a range \( 9.8 < p < 166.6 \text{ [bar]} \) and using \( G \) as parameter with values \( G = 750, 1000, 1500 \text{ and } 2000 \text{ [kg m}^{-2} \text{ s}^{-1}] \) for boiling upflow of water in a uniformly heated, vertical tube with 8 mm internal diameter. Give the value of the pressure for which \( x_{\text{crit}} \) becomes maximum. Use this pressure to plot \( x_{\text{crit}} \) as a function of \( G \).

**Solution:**

```scilab
// Exercise 28 - solved with Scilab
// Calculate p where Xcr is maximum
// b=4.08^2-4*3*0.68*1.57;
pr1 = (4.08-sqrt(b))/6/0.68;
G = 750:50:3000;
xcrit=[];
```
for i=1:length(G)
    gr = G(i)/1000;
    xcrit(i) = (0.39 + 1.57*pr1 - 2.04*pr1^2 + 0.68*pr1^3)/sqrt(gr);
end

xbasc();
xset("font size",14);
xset("thickness",2);
plot(G,xcrit)
xtitle("Xcr vs mass flux at p=51 bar","Mass flux, kg/m^2/s","Xcr")
xset("thickness",0);
xgrid(4)

**Answer:**

**Exercise 28a:** Calculate critical quality using the Levitan and Lantsman dryout correlation in a pipe with diameter $D = 12$ mm. The water coolant in the pipe is under 70 bar pressure and the mass flux is $G = 1200$ kg m$^{-2}$ s$^{-1}$.

At what distance from the inlet dryout will occur if the pipe is uniformly heated with heat flux $q'' = 1$ MW m$^{-2}$ and the inlet quality is -0.05? Assume $H_{fg} = 1505$ kJ kg$^{-1}$
Solution:

Exercise 28b: In a vertical pipe with D = 12 mm, cooled with water at 155 bar pressure and mass flux G = 2000 kg m^{-2} s^{-1}, calculate DNB heat flux at z = 1.0 m distance from the inlet. Use the Levitan and Lantsman correlation, Eq. (8.3.4). The pipe is heated with uniform heat flux q'' = 2 MW m^{-2} and the inlet water quality is -0.4.

Exercise 29: Calculate the heat transfer coefficient from the Groeneveld correlation (Eq. 8.4.5) for steam-water flow in a vertical tube: D = 10 mm, G = 1200 kg m^{-2} s^{-1}, p = 70 bar, heat flux q = 1.0 MW m^{-2} at axial positions where x = 0.2, 0.4, 0.6 and 0.8. Assume that the vapor Prandtl number varies with the temperature as Pr(T) = 1.05 -8*(T-623)*10^{-4}, where T is in [K]. Check whether the validity ranges are satisfied.

Exercise 30: Plot void fraction in function of flow quality (in a range from 0 to 1) assuming flow of water and vapor mixture at saturation conditions and at pressure p = 70 bar. Compare two cases: in one case both phases have the same averaged velocity; in the other case the vapor phase flows with mean velocity which is 20% higher than the mean liquid velocity.

Exercise 31: Compare void fraction predicted from the homogeneous model and the drift-flux model for steam-water flow. Assume p = 70 bar, G = 1200 kg m^{-2} s^{-1} and Eqs (8.5.2-11) and (8.5.2-12) for drift-flux parameters. Plot void fraction in function of quality. Plot slip ratio given by Eq. (8.5.2-14) in function of quality for the same conditions.
**Exercise 31a:** Boiling water is flowing with $G = 1200 \ \text{kg} \ \text{m}^{-2} \ \text{s}^{-1}$ in a pipe 12 mm ID at 70 bar pressure with the inlet quality equal to -0.05. The pipe is uniformly heated with $q'' = 1 \ \text{MW} \ \text{m}^{-2}$. Calculate the local value of the void fraction at $z = 0.5$ m distance from the inlet using the following models:

(a) HEM  
(b) DFM

**Exercise 32:** Plot two-phase multiplier (8.6.20) as a function of quality assuming two-phase flow of water-vapor mixture under 70 bar pressure.

**Exercise 33:** Plot two-phase multipliers (8.6.16) using three different definitions of mixture viscosity. Assume same conditions as in Exercise 32.

**Exercise 34:** Derive expressions for integral multipliers $r_2$, $r_3$ and $r_4$ for channels with saturated liquid at the inlet and with uniform heat flux distribution along channel length. Hint: in integrals along channel length use substitution: $dz = \text{const} * dx$, which is valid for uniformly heated channels.

**Exercise 35:** Solve Exercise 23 assuming constant heat flux applied to the channel and equal to $3 \ \text{MW} \ \text{m}^{-2}$.

**Home Assignment # 4**

Vertical smooth pipe with 8 mm internal diameter and length 3.6 m is uniformly heated with heat flux $q'' = 0.75 \ \text{MW} / \text{m}^2$. The pipe is cooled with water flowing vertically upwards, with inlet pressure 150 bar, inlet subcooling 70 K and mass flux $1000 \ \text{kg} \ \text{m}^{-2} \ \text{s}^{-1}$. Use saturated water properties at 150 bar pressure.

**Problem 1 (5 points):** Using the Bowring’s model for the onset of nucleate boiling and JensLottes correlation for the subcooled boiling predict the locations of the Onset of Nucleate Boiling ($z_{nb}$) and the Fully Developed Boiling ($z_{fdb}$) in the channel.

**Problem 2 (5 points):** Using the homogeneous equilibrium model (HEM) calculate the total pressure drop in the channel.

**Solution:**

```scilab
// // Home assignment #4 - solution with Scilab // D = 0.008; pi = 4*atan(1); PH = pi*D; A = pi*D*D/4; L = 3.6; G = 1000; p = 150; dTsubi = 70;
```
\[ q_{2p} = 0.75 \times 10^6; \]

\[ cp = 8512.98; \text{ // cplsat}(p); \]
\[ \text{visc} = 6.95 \times 10^{-5}; \text{ // vislsat}(p); \]
\[ \text{VISL} = \text{visc}; \]
\[ \text{VISG} = 2.2794 \times 10^{-5}; \text{ // visvsat}(p); \]
\[ \text{con} = 0.464; \text{ // conlsat}(p); \]
\[ \text{HFG} = 1.0; \text{ // hfg}(p); \]
\[ \text{RHOG} = 96.73; \text{ // rhovsat}(p); \]
\[ \text{RHOL} = 603.52; \text{ // rholsat}(p); \]

\[ \text{Pr} = \frac{cp \times \text{visc}}{\text{con}}; \]
\[ \text{Re} = \frac{G \times D}{\text{visc}}; \]

\[ \text{Solution} \]
\[ \text{Nu} = 0.023 \times \text{Re}^{0.8} \times \text{Pr}^{0.33}; \]
\[ h = \frac{\text{con} \times \text{Nu}}{D}; \]
\[ dT_{sub ONB} = \frac{q_{2p}}{h} - 25 \times (q_{2p}/10^6)^{0.25} \times \exp(-p/62); \]
\[ z_{ONB} = \frac{cp \times G \times A \times (dT_{subi} - dT_{sub ONB})}{q_{2p}/PH}; \]
\[ dT_{sub FDB} = \frac{q_{2p}/h}{1.4} - 25 \times (q_{2p}/1.4/10^6)^{0.25} \times \exp(-p/62); \]
\[ z_{FDB} = \frac{cp \times G \times A \times (dT_{subi} - dT_{sub FDB})}{q_{2p}/PH}; \]
\[ z_{SC} = \frac{cp \times G \times A \times dT_{subi}}{q_{2p}/PH}; \]

\[ \text{write}(\%io(2),z_{ONB},'(\text{''Non-boiling length = ''},f10.3,\text{'' m''})'); \]
\[ \text{write}(\%io(2),z_{FDB},'(\text{''Fully-developed boiling = ''},f10.3,\text{'' m''})'); \]
\[ \text{write}(\%io(2),z_{SC},'(\text{''Sub-cooled length = ''},f10.3,\text{'' m''})'); \]

\[ \text{// Pressure calculations} \]
\[ xi = -cp \times dT_{subi}/HFG; \]
\[ xe = xi + q_{2p} \times PH \times G/A/HFG; \]
\[ ae = 1/(1+RHOG/RHOL*(1-xe)/xe); \]
\[ C_{flo} = 0.0791 \times \text{Re}^{(-0.25)}; \]

\[ \text{// Pressure drop in single phase part} \]
\[ Lsp = z_{SC}; \]
\[ \text{dpFricSp} = -C_{flo} \times 2 \times Lsp \times G \times G/D/RHOL; \]
\[ \text{dpGravSp} = -RHOL \times 9.81 \times Lsp; \]
\[ \text{dpTotSp} = \text{dpFricSp} + \text{dpGravSp}; \]

\[ \text{// Pressure drop in two phase part} \]
\[ r2 = xe \times xe \times RHOL/ae/RHOG + (1-xe) \times (1-xe)/(1-ae) -1; \]
\[ f = \text{VISL/VISG} - 1; \]
\[ g = \text{RHOL/RHOG} - 1; \]
\[ c = G \times A \times HFG/q_{2p}/PH; \]
Lb = L - zSC;

\[ r3 = \frac{4}{3} \left(1 - \frac{g}{f}\right) + \frac{4}{7} \frac{g}{f} \left(1 + f \cdot xe\right) \cdot \frac{c}{Lb/f} \left(1 + f \cdot xe\right)^{\frac{3}{4}} - \left(\frac{4}{3} \left(1 - \frac{g}{f}\right) + \frac{4}{7} \frac{g}{f}\right) \cdot \frac{c}{Lb/f}; \]

a = \frac{RHOG}{RHOH};
b = 1 - \frac{RHOL - RHOG}{RHOH/Lb} \cdot \frac{xe/b + a/b \cdot \log(a/(b \cdot xe + a))}{a/b \cdot \log(a/(b \cdot xe + a))};
dpFricTp = -r3 \cdot Cflo \cdot 2 \cdot Lb \cdot G/G/D/RHOH;
dpGravTp = -r4 \cdot Lb \cdot RHOL \cdot 9.81;
dpAccTp = -r2 \cdot G/G/RHOH;

dpTotTp = dpFricTp + dpGravTp + dpAccTp;

dpTot = dpTotSp + dpTotTp;

// Total pressure drop if no boiling - for comparison

dpFricNb = -Cflo \cdot 2 \cdot L \cdot G/G/D/RHOH;
dpGravNb = -RHOL \cdot 9.81 \cdot L;

dpTotNb = dpFricNb + dpGravNb;

write(%io(2), dpFricSp, '(''Friction single-phase pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpGravSp, '(''Gravity single-phase pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpTotSp, '(''Total single-phase pressure drop = '', f10.1, '' Pa'')')
write(%io(2), r2, '(''Acceleration multiplier r2 = '', f10.3)')
write(%io(2), r3, '(''Friction multiplier r3 = '', f10.3)')
write(%io(2), r4, '(''Gravity multiplier r4 = '', f10.3)')
write(%io(2), dpFricTp, '(''Friction pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpGravTp, '(''Gravity pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpAccTp, '(''Acceleration pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpTotTp, '(''Total two-phase pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpTot, '(''Total channel pressure drop = '', f10.1, '' Pa'')')
write(%io(2), dpTotNb, '(''Non-boiling channel pressure drop = '', f10.1, '' Pa'')')

**Answer:** output from the program

Non-boiling length = 0.584 m
Fully-developed boiling = 0.880 m
Sub-cooled length = 1.589 m
Friction single-phase pressure drop = -2826.8 Pa
Gravity single-phase pressure drop = -9408.3 Pa
Total single-phase pressure drop = -12235.1 Pa
Acceleration multiplier r2 = 3.951
Friction multiplier r3 = 2.540
Gravity multiplier r4 = 0.777
Friction pressure drop = -9087.2 Pa
Gravity pressure drop = -9248.6 Pa
Acceleration pressure drop = -6546.3 Pa
Total two-phase pressure drop = -24882.1 Pa
Total channel pressure drop = -37117.2 Pa
Non-boiling channel pressure drop = -27717.9 Pa

Solution:
// Exercise 36 – solution with Scilab

D = 0.01;
LH = 3.6;

// Working conditions
p = 140.;
G = 2000;
q2p0 = 0.54203;  // Guess the value until MDNBR = 1.3

// properties
HFG = 1.0699e6;
HLIN = 1.3766e6;
HLSAT = 1.5710e6;

Xin = (HLIN-HLSAT)/HFG;

//
pi = atan(1)*4;

qcon = (10.3 - 7.8*p/98. + 1.6*(p/98)^2);  // Quality-independent part
qcon = qcon*sqrt(8/D/1000);  // Diameter correction

z = 0:0.1:3.6;
for i=1:length(z)
    q2p(i) = q2p0*(1.+cos(pi*(z(i)/LH-0.5)));
    x(i) = Xin + (4.*q2p0*1.e6/(G*D*HFG)) * (z(i)+LH/pi* (sin(pi*(z(i)/LH-0.5)) +1.));

}
q2pcr(i) = qcon*(G/1000.)^(1.2*((0.25*(p-98)/98)-x(i)))*exp(-1.5*x(i));
    dnbr(i) = q2pcr(i)/q2p(i);
end

// Plot the axial distributions of heat flux, critical heat flux
// DNBR and quality

plot(z,q2p,'o-',z,q2pcr,'^-.',z,dnbr,'v--',z,x,'k*','grid','on')
exlabel('Distance, [m]')
ylabel('Heat Flux, [MW/m^2], Quality [-]')
legend('Heat Flux','CHF','DNBR','Quality',1)

[MDNBR,k] = min(dnbr);

write(%io(2),q2p0,    (''q2p0          = '',f10.4,'' MW/m^2''))
write(%io(2),MDNBR,   (''MDNBR         = '',f10.4)'')
write(%io(2),z(k),    (''At distance z = '',f10.4,'' m from the
             inlet'')')
write(%io(2),q2pcr(k),(''CHF           = '',f10.4,'' MW/m^2''))
write(%io(2),q2p(k),  (''Local h. flux = '',f10.4,'' MW/m^2''))

**Answer:** output from the program

MDNBR   =  1.3000
At distance z =  2.8000 m from the inlet
CHF     =  1.1576 MW/m^2
Local h. flux =  0.8904 MW/m^2