Outline of the Lecture

• Application of the Effective Multiplication Factor
• Reactivity
• Units of Reactivity
• Reactivity Coefficients and Reactivity Defects
• Moderator Effects
• Moderator Temperature Coefficient
• Fuel Temperature Coefficient
• Pressure Coefficient
• Void Coefficient
Application of the Effective Multiplication Factor

• When $k_{\text{eff}}$ remains constant from generation to generation, it is possible to determine the number of neutrons beginning any particular generation by knowing only the value of $k_{\text{eff}}$ and the number of neutrons starting the first generation

\[
\begin{array}{cccc}
N_0 & N_0 \cdot k_{\text{eff}} & N_0 \cdot (k_{\text{eff}})^2 & N_0 \cdot (k_{\text{eff}})^3 \\
\text{Gen: 0} & 1 & 2 & 3 & n \\
\end{array}
\]

• Thus, after $n$-generations, the initial number of neutrons $N_0$ will be $N_n = N_0 \cdot (k_{\text{eff}})^n$
Reactivity (1)

- If there are $N_0$ neutrons in the preceding generation, then there are $N_0 * k_{\text{eff}}$ neutrons in the present generation.

- The gain or loss in the neutron population $N_0 * k_{\text{eff}} - N_0$ expressed as a fraction of the present generation $N_0 * k_{\text{eff}}$ is referred to as reactivity $\rho$.

$$\rho = \frac{N_0 k_{\text{eff}} - N_0}{N_0 k_{\text{eff}}} = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$$
Units of Reactivity (1)

• Reactivity is a dimensionless number

• However, the value of radioactivity is often a small decimal value

• In order to make this value easier to express, artificial units are defined
Units of Reactivity (2)

- By definition, the value for reactivity is in units of $\Delta k/k$

- Alternative units for reactivity are $\% \Delta k/k$ and pcm (percent millirho)

- The conversion between these units are as follows

  $1 \Delta k/k = 100\% \Delta k/k = 10^5$ pcm
  $1\% \Delta k/k = 0.01 \Delta k/k$
  $1$ pcm $= 10^{-5} \Delta k/k$
Units of Reactivity (3)

• Example:

Calculate the reactivity in the reactor core when $k_{\text{eff}}$ is equal to 1.002 and 0.998

• Solution:

the reactivity is as follows:

\[
\rho = \frac{(k_{\text{eff}}-1)}{k_{\text{eff}}} = \frac{(1.002-1)}{1.002} = 0.001996 \quad \frac{\Delta k}{k} = 0.1996\% \Delta k/k = 199.6 \text{ pcm}
\]

\[
\rho = \frac{(k_{\text{eff}}-1)}{k_{\text{eff}}} = \frac{(0.998-1)}{0.998} = -0.002 \quad \frac{\Delta k}{k} = -0.2\% \Delta k/k = -200 \text{ pcm}
\]
Units of Reactivity (4)

• Other units often used in reactor analyses are dollars ($) and cents

• These units will be motivated in detail in Lecture 5 dealing with the reactor kinetics

• One dollar (1$) reactivity is equivalent to the effective delayed neutron fraction

• One cent (1c) reactivity is equal to one-hundredth of a dollar
Reactivity Coefficients and Reactivity Defects (1)

• The amount of reactivity in a reactor core determines the time change of the neutron population and thus the reactor power

• The reactivity can be affected by several factors
  – Fuel depletion
  – Temperature
  – Pressure
  – Poisons
  – Control rod insertion
  – Etc…
Reactivity Coefficients and Reactivity Defects (2)

• **Reactivity coefficients** are used to quantify the effect of variation in parameters on the reactivity of the core.

• Reactivity coefficients are the amount that the reactivity will change for a given change in the parameter.

• For instance, the increase in moderator temperature will cause a decrease in the reactivity of the core.
Reactivity Coefficients and Reactivity Defects (3)

- The amount of reactivity change per degree change in moderator temperature is the moderator temperature coefficient.

- Typical units for moderator temperature coefficient are pcm/K.

- Reactivity coefficients are typically symbolized by \( \alpha_x \), where \( x \) represents some variable parameter that affects reactivity (e.g. Temperature).
Reactivity Coefficients and Reactivity Defects (4)

• The reactivity coefficient can be expressed as a derivative of reactivity against the given parameter

• For example: the temperature coefficient of reactivity is as follows

\[ \alpha_T = \frac{\partial \rho}{\partial T} = \frac{\partial}{\partial T} \left( \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \right) = \frac{1}{k_{\text{eff}}^2} \frac{\partial k_{\text{eff}}}{\partial T} \approx \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial T} \]

The last approximation is valid since \( k_{\text{eff}} \) is close to 1
Reactivity Coefficients and Reactivity Defects (5)

- We can develop further this equation by recalling the six-factor formula: \( k_{\text{eff}} = k_\infty * P_{FNL} * P_{TNL} \) from which we get

\[
\ln k_{\text{eff}} = \ln k_\infty + \ln P_{FNL} + \ln P_{TNL}
\]

- Differentiating both sides of this equation leads to the following expression for the temperature reactivity coefficient

\[
\alpha_T \approx \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial T} = \frac{1}{k_\infty} \frac{\partial k_\infty}{\partial T} + \frac{1}{P_{FNL}} \frac{\partial P_{FNL}}{\partial T} + \frac{1}{P_{TNL}} \frac{\partial P_{TNL}}{\partial T}
\]
Reactivity Coefficients and Reactivity Defects (6)

• In a similar way, using the four factor formula, we can find

\[
\frac{1}{k_\infty} \frac{\partial k_\infty}{\partial T} = \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} + \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T} + \frac{1}{p} \frac{\partial p}{\partial T}
\]

• It can be seen now that the temperature reactivity coefficient can be found from known dependence of all factors in the six factor formula on the temperature

• The same procedure can be used to calculate the other reactivity coefficients, e.g. void or/and pressure
Reactivity Coefficients and Reactivity Defects (7)

- Reactivity defect ($\Delta \rho$) is the total reactivity change caused by a variation in a parameter.

- Reactivity defect is determined by multiplying the change in a parameter ($\Delta x$) by the average value of the reactivity coefficient for that parameter, $\alpha_x$:

$$\Delta \rho = \alpha_x \Delta x$$
Reactivity Coefficients and Reactivity Defects (8)

• Example:
  the moderator coefficient for a reactor is -16 pcm/K. Calculate the reactivity defect that results from a temperature decrease of 2.5 K.

• Solution:
  \[ \Delta \rho = \alpha_T \Delta T = (-16 \text{ pcm/K}) \times (-2.5 \text{ K}) = 40 \text{ pcm} \]

  thus the reactivity addition due to the temperature decrease was positive because of the negative temperature coefficient
Moderator Effects (1)

• A good moderator should possess the following desirable characteristics:
  – Large neutron scattering cross section
  – Low neutron absorption cross section
  – Large neutron energy loss per collision

• The major reactor types that are currently employed use moderating materials to reduce fission neutron energies to the thermal range

• Light moderators are more effective than heavy moderators since light nuclei removes more neutron energy per collision (leading to higher resonance escape probability)
Moderator Effects (2)

- The ability of a given material to slow down neutrons is referred to as the **macroscopic slowing down power** (MSDP) and is defined as:

\[
\text{MSDP} = \xi \Sigma_s
\]

here \(\xi\) is the logarithmic energy decrement per collision and \(\Sigma_s\) is the macroscopic scattering cross section for neutrons.
Moderator Effects (3)

- Macroscopic slowing down power MSDP indicates how rapidly slowing down occurs, however, it does not define how effective moderator material is.

- An element such as boron has good MSDP but it is a poor moderator because of its high probability of absorbing neutrons.

- A better measure of moderating property is the moderating ratio (MR) defined as:

\[ MR = \frac{MSDP}{\Sigma_a} = \frac{\xi \Sigma_s}{\Sigma_a} \]
Moderator Effects (4)

• Another ratio, the moderator-to-fuel ratio \( (N^m/N^U) \) is very important parameter in discussion of moderators.

• As the reactor designer increases the amount of moderator in the core, neutron leakage decreases, neutron absorption in the moderator increases and causes a decrease in the thermal utilization factor.

• Decreasing the amount of moderator causes an increase in slowing down time and results in a greater loss of neutrons by resonance absorption and causes an increase in neutron leakage.
Moderator Effects (5)

• Because the moderator-to-fuel ratio affects the thermal utilization factor and the resonance escape probability, it also affects $k_{\text{eff}}$

• In other words, $k_{\text{eff}}$ is a function of $(N_{m}/N_{U})$

• The dependence $k_{\text{eff}} = f (N_{m}/N_{U})$ is illustrated in a figure on the next slide
Moderator Effects (6)

- Resonance escape probability
- Thermal utilization factor

Under moderated | Over moderated

$k_{\text{eff}}$
Moderator Effects (7)

• As seen in the figure, there is an optimum point above which increasing the moderator-to-fuel ratio decreases $k_{\text{eff}}$ due to the dominance of decreasing thermal utilization factor.

• Below this point a decrease in the moderator-to-fuel ratio decreases $k_{\text{eff}}$ due to the dominance of the increased resonance absorption in the fuel.

• Core is said to be under moderated for $(N^m/N^U)$ below the optimum and over moderated otherwise.
Moderator Effects (8)

• In practice water-moderated reactors are operated in the under moderated region.

• In that way a reactor is more self-regulating since an increase in temperature will cause decrease in \((N^m/N^U)\) and this will lead to additional decreasing of \(k_{\text{eff}}\).

• For over moderated reactors the effect would be opposite and increase of temperature would lead to further increase of \(k_{\text{eff}}\).
Moderator Temperature Coefficient (1)

- The change in reactivity per degree change in temperature is called the temperature coefficient of reactivity.

- Because different materials in the reactor have different temperatures during reactor operation, several different temperature coefficients are used.

- Usually the two dominant temperature coefficients are the **moderator temperature coefficient** and the **fuel temperature coefficient**.
Moderator Temperature Coefficient (2)

• the change in reactivity per degree change in moderator temperature is called the **moderator temperature coefficient** (also **delayed temperature coefficient**)

• The magnitude and sign (+ or -) of the moderator temperature coefficient is primarily a function of moderator-to-fuel ratio:
  – If a reactor is under moderated it will have a negative moderator temperature coefficient

• Negative moderator temperature coefficient is desirable because of its self-regulating effect
Example: moderator temperature coefficient for a BWR reactor.
Moderator Temperature Coefficient (4)

- As shown in figure, the moderator temperature coefficient is about -5 to -10 pcm/K at room temperature, but it decreases to ca -25 pcm/K for operating temperature (286 C)

- At the end of cycle the coefficient can be slightly positive, about +5 pcm/K
Moderator Temperature Coefficient (5)

- Of all six factors in the six factor formula, only $p$ (resonance escape probability) and $f$ (thermal utilization factor) indicate a significant dependence on the temperature.

- Since only $f$ factor is dependent on moderator properties, the temperature reactivity coefficient for moderator is:

$$\alpha_{T,m} = \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} + \frac{1}{p} \frac{\partial p}{\partial T} + \frac{1}{\epsilon} \frac{\partial \epsilon}{\partial T} + \frac{1}{P_{FNL}} \frac{\partial P_{FNL}}{\partial T} + \frac{1}{P_{TNL}} \frac{\partial P_{TNL}}{\partial T} \approx \frac{1}{f} \frac{\partial f}{\partial T}$$
Moderator Temperature Coefficient (6)

• The thermal utilization factor can be written as

\[
f = \frac{\Sigma^U_a}{\Sigma^U_a + \Sigma^m_a + \Sigma^p_a + \Sigma^c_a}
\]

• If now with increased moderator temperature its density decreases, the macroscopic cross section of moderator will also decrease, since

\[
\Sigma^m_a = \frac{10^3 \rho_m N_A}{M} \sigma^m_a
\]

• It means that the thermal utilization factor will increase with increasing moderator temperature (assuming that all other parameters are not dependent on the temperature)
Fuel Temperature Coefficient (1)

• Another temperature coefficient – the fuel temperature coefficient – has a greater effect than the moderator temperature coefficient for some reactors.

• The **fuel temperature coefficient** is the change in reactivity per degree change in fuel temperature.

• The coefficient is also called the **prompt temperature coefficient** because an increase in reactor power causes an immediate change in fuel temperature.
Fuel Temperature Coefficient (2)

- A negative fuel temperature coefficient is generally considered to be even more important than a negative moderator temperature coefficient because fuel temperature immediately increases following an increase in reactor power.

- The time for heat to be transferred to moderator is measured in seconds.

- In event of a large positive reactivity insertion, the moderator temperature cannot turn the power rise for several seconds, whereas the fuel temperature coefficient starts adding negative reactivity immediately.
Fuel Temperature Coefficient (3)

• Another name applied to the fuel temperature coefficient of reactivity is the **fuel doppler reactivity coefficient**

• The name is applied because in typical low enrichment, light water moderated, thermal reactors the fuel temperature coefficient of reactivity is negative and is the result of the doppler effect, also called doppler broadening
Fuel Temperature Coefficient (4)

- In the similar manner as for the moderator temperature coefficient it can be shown that

\[ \alpha_{T,F} = \frac{1}{\eta} \frac{\partial \eta}{\partial T} + \frac{1}{f} \frac{\partial f}{\partial T} + \frac{1}{p} \frac{\partial p}{\partial T} + \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial T} + \frac{1}{P_{FNL}} \frac{\partial P_{FNL}}{\partial T} + \frac{1}{P_{TNL}} \frac{\partial P_{TNL}}{\partial T} \approx \frac{1}{p} \frac{\partial p}{\partial T} \]

- Assuming \( p = \exp \left[ - \frac{N_F \cdot I}{\xi \cdot \Sigma_s} \right] \)

- Then

\[ \alpha_{T,F} = \frac{1}{p} \frac{\partial p}{\partial T} = \left( - \frac{N_F}{\xi \Sigma_s} \right) \frac{\partial I}{\partial T} = - \frac{N_F}{\xi \Sigma_s} \frac{I(300K)\beta}{2\sqrt{T}} \]

- Since

\[ I(T) = I(300 \text{ K}) \left[ 1 + \beta \left( \sqrt{T} - \sqrt{300} \right) \right] \]
Pressure Coefficient (1)

• The reactivity in a reactor core can be affected by the system pressure

• The *pressure coefficient of reactivity* is defined as the change in reactivity per unit change in pressure

• The pressure coefficient of reactivity for the reactor is the result of the effect of pressure on the density of the moderator

• This coefficient is small och seldom a major factor
Pressure Coefficient (2)

• In systems with boiling conditions, such as boiling water reactors (BWR), the pressure coefficient becomes an important factor due to the larger density changes that occur when the vapor phase of water undergoes a pressure change
Void Coefficient (1)

- Void fraction is defined as the fraction of vapor in a certain volume of two-phase mixture

- It can be written as follows

\[ \alpha = \frac{\text{Volume of vapor}}{\text{Total volume}} = \frac{V_v}{V} \]

- Note that for the void fraction is used the same symbol as for the reactivity coefficients, but they should not be confused! The reactivity coefficient always have subscript indicating variable parameter, e.g. \( \alpha_T \) is the temperature coefficient of reactivity
Void Coefficient (2)

• The void coefficient of reactivity is defined as the change in reactivity per percent change in void volume

• The void coefficient is significant in water moderated reactors that operate at near saturated conditions

• Since steam density is much lower than the one of water, an increase in the steam content in the core will decrease the number of neutrons reaching the thermal energies, decreasing the number of fissions and thus the reactor power
Void Coefficient (3)

• The void coefficient of reactivity is defined as

\[ \alpha_v = \frac{1}{k_{\text{eff}}} \frac{\partial k_{\text{eff}}}{\partial \alpha} \]

• The coefficient can be calculated by considering the effect of changes of void fraction on the different factors of \( k_{\text{eff}} \)

• A typical value the void coefficient of reactivity in BWRs is

\[ \alpha_v \approx -0.12 \]
Exercises (1)

• **Exercise 3:** Calculate the thermal utilization factor for a homogenized core composed of (in % by volume): UO₂ 35% and H₂O 65%. The enrichment of the fuel is 3.2% (by weight). Microscopic cross sections [b] for absorption are as follows: water 0.66 [b], oxygen O: 2x10⁻⁴ [b], U-235: 681 [b], U-238: 2.7 [b].

Density of UO₂: 10200 kg/m³
Density of water: 800 kg/m³

How the thermal utilization factor changes if the water density increases to 900 kg/m³?
Exercises (2)

• **Exercise 4:** Calculate the moderating power and the moderating ratio for H₂O (density 1000 kg/m³) and Carbon (density 1600 kg/m³). The macroscopic cross sections are given below:

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Microscopic cross sections [b]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>absorption</td>
<td>scattering</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>0.332</td>
<td>38</td>
</tr>
<tr>
<td>Oxygen</td>
<td>27 x 10⁻⁵</td>
<td>3.76</td>
</tr>
<tr>
<td>Carbon</td>
<td>0.0034</td>
<td>4.75</td>
</tr>
</tbody>
</table>
Exercises (3)

• **Exercise 5:** Calculate the resonance escape probability for a reactor as in Exercise 3 assuming the fuel temperature \( T = 1500 \text{ K} \) and the effective resonance integral for fuel at \( T = 300 \text{ K} \) equal to 25 [b]. Microscopic cross sections for scattering are as follows: water 103 [b], oxygen O: 6 [b], U-235: 8 [b], U-238: 8.3 [b].
Exercises (4)

• **Exercise 6:** Calculate the fuel reactivity coefficient for a reactor as in Exercise 3 assuming the fuel temperature $T = 1500$ K and the effective resonance integral for fuel at $T = 300$ K equal to 25 [b]. Microscopic cross sections for scattering are as follows: water 103 [b], oxygen O: 6 [b], U-235: 8 [b], U-238: 8.3 [b].