8. Thermal-Hydraulic Analysis of Two-Phase Flows in Heated Channels

This section is dealing with the thermal-hydraulic analysis of two-phase flows in heated channels. Figure 8.0.1 shows the various two-phase flow patterns encountered over the length of a heated channel, together with the corresponding heat transfer regions.

![Diagram showing flow patterns and heat transfer regions](image)

Figure 8.0.1. Flow patterns and regions of heat transfer in convective boiling.
Several of the heat transfer regions shown in Fig. 8.0.1 are encountered in fuel assemblies of LWRs. During normal operation of a nuclear power plant the following regions are present: convective heat transfer to liquid, subcooled boiling, saturated nucleate boiling and forced convective heat transfer through liquid film. During abnormal operation two other regions can occur: post-dryout heat transfer and convective heat transfer to vapor. Details of the various heat transfer regimes will be given in the following sections. In addition, two types of Critical Heat Flux (CHF) will be discussed in some detail.

### 8.1 Onset of Nucleate Boiling

In Chapter 7 it has been shown that the coolant temperature in a channel heated with a uniform heat flux is a linear function of the distance from the inlet as follows,

\[
T_f(z) = T_{fi} + \frac{q''P_H}{c_p G A} z. \tag{8.1.1}
\]

The saturated nucleate boiling starts at \( z = z_{sc} \), where the bulk liquid temperature \( T_f(z) \) equals \( T_s \), the saturation temperature. The length of the subcooled region \( z_{sc} \) can be readily obtained from Eq. (8.1.1) as,

\[
z_{sc} = \frac{c_p G A}{q'' P_H} (T_s - T_{fi}) = \frac{c_p G A}{q'' P_H} \Delta T_{sub} , \tag{8.1.2}
\]

where \( \Delta T_{sub} \) is the so-called inlet subcooling.

Assume that the subcooled boiling start at \( z = z_{nb} \) (\( z_{nb} \) is the length of non-boiling region). In the single-phase region, when \( 0 < z < z_{nb} \), the clad surface temperature \( T_C \) and the liquid bulk temperature \( T_f \) are related to each other as follows,

\[
T_C - T_f \equiv \Delta T_f = \frac{q''}{h} , \tag{8.1.3}
\]

where \( h \) is the heat transfer coefficient to single phase liquid under forced convection and \( \Delta T_f \) is the temperature difference between the surface of the heated wall and the bulk liquid.
Figure 8.1.1. Wall and liquid temperature distribution in subcooled boiling.

Combining Eq. (8.1.1) and (8.1.3) yields the following expression for the clad surface temperature,

$$T_C(z) = T_f(z) + \frac{q''}{h} = T_f + q'' \left( \frac{P_H z}{c_p GA} + \frac{1}{h} \right).$$  \hspace{1cm} (8.1.4)

It is convenient to introduce a so-called wall superheat defined as a difference of the temperature of the heated wall surface and the saturation temperature. The wall-superheat can be obtained from Eq. (8.1.4) as,

$$\Delta T_{sup}(z) \equiv T_C(z) - T_s = -\Delta T_{sub} + q'' \left( \frac{P_H z}{c_p GA} + \frac{1}{h} \right).$$  \hspace{1cm} (8.1.5)

It is clear that no boiling will occur when $\Delta T_{sup} < 0$, since the wall temperature is below the saturation temperature. However, there is no clear and easy criterion to define such wall superheat at which the subcooled boiling will start. This quantity has been investigated by many researches. Bergles and Roshenow proposed the following correlation for the Onset of Nucleate Boiling (ONB),

$$q''_{ONB} = 1083 \cdot P^{1.156} \cdot \left( \frac{1.8 \Delta T_{sup}}{P^{0.0264}} \right)^{2.16}.$$  \hspace{1cm} (8.1.6)
where $q''_{ONB}$ is the heat flux at onset-of-nucleate-boiling [W m$^{-2}$], $p$ is pressure [bar] and $\Delta T_{sup}$ is wall superheat [K].

Another model was proposed by Davies and Anderson, who proposed the following analytical solution,

$$q''_{ONB} = \frac{H_{fg} \rho_f \rho_g \lambda_f}{8 \sigma_s (\rho_f - \rho_g)} \Delta T_{sup}^2. \quad (8.1.7)$$

Here $\sigma$ is the surface tension [N m$^{-1}$], $H_{fg}$ is the latent heat [J kg$^{-1}$] and $\lambda_f$ is the liquid conductivity [W K$^{-1}$ m$^{-1}$].

Combining Eqs. (8.1.6) or (8.1.7) with (8.1.5) yields the distance $z_{nb}$ at which the subcooled boiling starts.

Bowring proposed another approach, based on an arbitrary assumption that the non-boiling length $z_{nb}$, or the onset-of-nucleate-boiling point is located where the wall temperature calculated from single-phase convective heat transfer would be equal to the wall temperature calculated from the subcooled boiling heat transfer, that is

$$(T_C)_{SPE} = (T_C)_{SCB}, \quad (8.1.8)$$

or

$$T_f(z) + \frac{q''}{h} = T_s + \psi \cdot (q^{*})^n. \quad (8.1.9)$$

Bowring’s expression for the local subcooling for the onset of nucleate boiling is thus as follows

$$\Delta T_{sub}(z)_{ONB} = \frac{q''}{h} - \psi \cdot (q^{*})^n \quad (8.1.10)$$

The right-hand-side of Eq. (8.1.9) is a general form of a correlation for wall temperature in subcooled boiling heat transfer. Several such correlations have been published in the literature. One example is the Jens and Lotes correlation, developed for subcooled boiling of water flowing upwards in vertical electrically heated stainless steel or nickel tubes. The correlation for the wall temperature is as follows

$$T_C = T_s + 25 \left( \frac{q''}{10^6} \right)^{0.25} e^{-p/62}. \quad (8.1.11)$$

It should be noted that this correlation is valid for water only and that it is dimensional: $T$ is in [K], $q''$ is in [W m$^{-2}$] and $p$ is absolute pressure in [bar].
More recently, Thom et al. reported that the wall superheat obtained from (8.1.11) was consistently under-predicted in comparison with their measurements. They proposed the following modified correlation

$$T_C = T_s + 22.65 \left( \frac{q''}{10^6} \right)^{0.5} e^{-p/87},$$  \hspace{1cm} (8.1.12)

and all variables are dimensional in a similar manner as in the Jens-Lotes correlation.

### 8.2 Subcooled and Saturated Flow Boiling

Subcooled boiling region is divided into two sub-regions: partial subcooled boiling and fully-developed subcooled boiling region. In the partial subcooled boiling region only few nucleation sites are active and a considerable portion of the heat is transferred by normal single-phase forced convection. When the wall temperature increases, the number of active sites also increases and the area for single-phase heat transfer decreases. As the wall temperature is increased further, the whole surface is covered by active nucleation sites and boiling starts to be fully-developed.

Bowring suggested the following heat flux partitioning in the partial boiling region,

$$q'' = q''_{spl} + q''_{sch},$$  \hspace{1cm} (8.2.1)

where $q''$ is the total average surface heat flux, $q''_{spl}$ is the average surface heat flux transferred by single-phase convection and $q''_{sch}$ is the average surface heat flux transferred by bubble nucleation.

The single-phase convection part $q''_{spl}$ can be calculated in the same way as in single-phase region, e.g. by using Eqs. (7.1.1) with (7.1.3). For the bubbly-nucleation part $q''_{sch}$, Rohsenow proposed a correlation derived for nucleate pool boiling,

$$q''_{sch} = \mu_f H_{fg} \left[ \frac{g(\rho_f - \rho_g)}{\sigma} \right]^{0.5} \left[ \frac{c_p \Delta T_{sup}}{C_{sf} H_{fg}} \right]^{1/r} \Pr^{-s/r},$$  \hspace{1cm} (8.2.2)

where $r = 0.33$ and $s = 1.0$ for water and 1.7 for other fluids. The constant $C_{sf}$ may vary from one fluid-surface combination to another. For water on mechanically polished stainless steel, the coefficient is equal to 0.0132.

Experimental data indicate that there is a correlation between the heat flux at the onset of nucleate boiling and the heat flux at which boiling becomes fully developed. This correlation has been approximated by Forster and Grief as
\[ q''_{FDB} = 1.4q''_{ONB} \]  \hspace{1cm} (8.2.3)

This observation leads to the following expression for the local subcooling at the fully developed boiling location, due to Bowring,

\[ \Delta T_{sub}(z)_{FDB} = \frac{q''}{1.4h} - \psi \left( \frac{q''}{1.4 \cdot 10^5} \right)^n \]  \hspace{1cm} (8.2.4)

Saturated flow boiling region occurs in fuel assemblies of BWRs where nearly complete vaporization of the coolant is desired. To avoid the high wall temperatures and/or the poor heat transfer associated with the saturated film-boiling regime, the vaporization must be accomplished at low superheat or low heat flux levels. When boiling is initiated, both nucleate boiling and liquid convection may be the active heat transfer mechanisms. The importance of the two mechanisms varies over the channel length. As vaporization occurs, void fraction rapidly increases causing flow acceleration and, as a result, an enhancement of the convective heat transfer.

The increasing void fraction and acceleration of flow causes changes in the flow pattern along the boiling channel. For vertical upward flow, the flow pattern changes from bubbly to slug, churn and the annular flow. Correspondingly, the heat transfer regime changes from nucleate flow boiling to evaporation of the liquid film in annular flow. Further, it is clear that, as vaporization continues, the thickness of the liquid film will decrease, reducing its thermal resistance and thereby enhancing the effectiveness of this mechanism. When the liquid film becomes very thin, the required superheat to transport the wall heat across the liquid film become so low that nucleation is completely suppressed, and the only heat transfer mechanism is heat conduction through the liquid film.

It should be clear that prediction of the convective boiling heat transfer coefficient requires an approach that accommodates a transition from a nucleate-pool-boiling-like condition at low qualities to a nearly pure film evaporation condition at higher qualities.

One of the most popular correlations for prediction of heat transfer coefficient in saturated flow boiling is the Chen correlation. The fundamental assumption is that the total heat transfer coefficient is a superposition of two parts: a microscopic (nucleate boiling) contribution \( h_{mic} \) and a macroscopic (bulk convective) contribution \( h_{mac} \):

\[ h = h_{mic} + h_{mac} \]  \hspace{1cm} (8.2.5)

The bulk-convective contribution is evaluated using the Dittus-Boelter equation,

\[ h_{mac} = 0.023 \left( \frac{\lambda_f}{D} \right) \text{Re}^{0.8} \text{Pr}^{0.4} \cdot F. \]  \hspace{1cm} (8.2.6)

Here liquid Reynolds number is defined as,
\[
\text{Re} = \frac{G(1-x)D}{\mu}.
\]  
(8.2.7)

\( S \) is the so-called suppression factor evaluated as,

\[
S = \left(1 + 2.56 \cdot 10^{-6} F^{1.463} \cdot \text{Re}^{1.17}\right)^{-1},
\]  
(8.2.8)

where \( F \) is found as,

\[
F = \begin{cases}
1 & X_{tt}^{-1} \leq 0.1 \\
2.35 \left(0.213 + \frac{1}{X_{tt}}\right) & X_{tt}^{-1} \geq 0.1
\end{cases}
\]  
(8.2.9)

and the Martinelli parameter is given by

\[
X_{tt} = \left(\frac{1-x}{x}\right)^{0.9} \left(\frac{\rho_g}{\rho_f}\right)^{0.5} \left(\frac{\mu_f}{\mu_g}\right)^{0.1}.
\]  
(8.2.10)

The microscopic contribution (nucleate boiling) to the overall heat transfer coefficient is determined by applying a correction to the Forster-Zuber relation for the heat transfer coefficient for nucleate pool boiling,

\[
h_{\text{mic}} = 0.00122 \left[ \frac{\lambda_f^{0.79} c_p^{0.45} \rho_f^{0.49}}{\sigma^{0.5} \mu_f^{0.29} H_f^{0.24} \rho_g^{0.24}} \right] \Delta T_{\text{sup}}^{0.24} \left(p_x(T_c) - p_f\right)^{0.75} \cdot \mu.
\]  
(8.2.11)

The Chen correlation has to be applied as follows:

1. Given mass flux, fluid properties, local heat flux and quality; guessed wall superheat
2. Find \( X_{tt} \) from Eq. (8.2.10)
3. Find Re from Eq. (8.2.7)
4. Find \( F \) from Eq. (8.2.9)
5. Find \( S \) from Eq. (8.2.8)
6. Find \( h_{\text{mac}} \) from Eq. (8.2.6)
7. Find \( h_{\text{mic}} \) from Eq. (8.2.11)
8. Find \( h \) from Eq. (8.2.5)
9. Find superheat as \( \Delta T_{\text{sup}} = q^* / h \)
10. Repeat 7 through 9 until convergence is achieved

### 8.3 Occurrence of Critical Heat Flux

The conditions at which the wall temperature rises and the heat transfer decreases sharply due to a change in the heat transfer mechanism are termed as the Critical Heat Flux (CHF) conditions. The nature of CHF, and thus the change of heat transfer mechanism, varies
with the enthalpy of the flow. At subcooled conditions and low qualities this transition corresponds to a change in boiling mechanism from nucleate to film boiling. For this reason the CHF condition for these circumstances is usually referred to as the Departure from Nucleate Boiling (DNB).

At saturated conditions, with moderate and high qualities, the flow pattern is almost invariably in an annular configuration (see Fig. 8.0.1). In these conditions the change of the heat transfer mechanism is associated with the evaporation and disappearance of the liquid film and the transition mechanism is termed as dryout. As indicated in Fig. 8.0.1 once dryout occurs, the flow pattern changes to the liquid-deficient region, with a mixture of vapor and entrained droplets. It is worth noting that due to high vapor velocity the heat transport from heated wall to vapor and droplets is quite efficient, and the associated increase of wall temperature is not as dramatic as in the case of DNB.

The mechanisms responsible for the occurrence of CHF (DNB- and dryout-type) are not fully understood, even though a lot of effort has been devoted to this topic. Since no consistent theory of CHF is available, the predictions of CHF occurrence rely on correlations obtained from specific experimental data. LWR fuel vendors perform their own measurements of CHF in full-scale mock-ups of fuel assemblies. Based on the measured data, proprietary CHF correlations are developed. As a rule, such correlations are limited to the same geometry and the same working conditions as used in experiments.

Most research on CHF published in the open literature has been performed for upward flow boiling of water in uniformly heated tubes. The overall experimental effort in obtaining CHF data is enormous. It is estimated that several hundred thousand CHF data points have been obtained in different labs around the world. More than 200 correlations have been developed in order to correlate the data. Discussion of all such correlations is not possible; however, some examples will be described in this section.

One of the earliest correlations was given by Bowring, who proposed the following expression to evaluate the CHF condition:

$$\dot{q}_{\text{crit}}^* = \frac{A + D \cdot G \left( H_{fs} - H_{cr} \right)}{C + z}, \quad (8.3.1)$$

where

$$A = \frac{0.579 F_{B1} D \cdot G \cdot H_{lg}}{1 + 0.0143 F_{B2} D^{1/2} G}, \quad (8.3.2)$$

$$C = \frac{0.077 F_{B3} D \cdot G}{1 + 0.347 F_{B4} \left( G / 1356 \right)^{2 - 0.00725 \rho}}, \quad (8.3.3)$$
Here \( D \) is the hydraulic diameter in \([\text{m}]\), \( G \) is the mass flux in \([\text{kg m}^{-2} \text{s}^{-1}]\), \( H_f \) is the latent heat in \([\text{J kg}^{-1}]\), \( p \) is the pressure in \([\text{bar}]\), \( H_{fs} \) is the enthalpy of saturated liquid at local pressure and \( H_{ci} \) is the inlet enthalpy. The correlation parameters \( F_{B1}, F_{B2}, F_{B3} \) and \( F_{B4} \) are functions of pressure and are given in a graphical form (not shown here, but these who are interested can found the plots in Liquid-Vapor Phase Change Phenomena, Van P. Carey, Hemisphere 1992). The correlation is based on a fit to data in the ranges \( 136 < G < 18600 \) \([\text{kg m}^{-2} \text{s}^{-1}]\), \( 2 < p < 190 \) \([\text{bar}]\), \( 2 < D < 45 \) \([\text{mm}]\) and \( 0.15 < z < 3.7 \) \([\text{m}]\).

For upflow boiling of water in vertical tubes with constant applied heat flux, Levitan and Lantsman recommended the following correlation for DNB in 8-mm-diameter pipe:

\[
q'^*_{\text{crit}} = \left[ 10.3 - 7.8 \frac{p}{98} + 1.6 \left( \frac{p}{98} \right)^2 \left( \frac{G}{1000} \right)^{1.2 \left[ 0.25(p-98) - 1 \right]} \right] e^{-1.5x}. \tag{8.3.4}
\]

For dryout they recommended the following relation for predicting the critical quality for 8-mm pipe:

\[
x_{\text{crit}} = \left[ 0.39 + 1.57 \frac{p}{98} - 2.04 \left( \frac{p}{98} \right)^2 + 0.68 \left( \frac{p}{98} \right)^3 \right] \frac{G}{1000}^{-0.5} \tag{8.3.5}
\]

In these relations \( q'^*_{\text{crit}} \) is in \([\text{MW m}^{-2}]\), \( p \) is the pressure in \([\text{bar}]\), \( G \) is the mass flux in \([\text{kg m}^{-2} \text{s}^{-1}]\). The correlation (8.3.4) is valid in ranges \( 29.4 < p < 196 \) \([\text{bar}]\) and \( 750 < G < 5000 \) \([\text{kg m}^{-2} \text{s}^{-1}]\) and is accurate to \( \pm 15\% \). For correlation (8.3.5) the ranges are \( 9.8 < p < 166.6 \) \([\text{bar}]\) and \( 750 < G < 3000 \) \([\text{kg m}^{-2} \text{s}^{-1}]\) and the accuracy of \( x_{\text{crit}} \) is \( \pm 0.05 \).

Both these correlations can be used for CHF in tubes with other diameters by using proper correction factors. The CHF given by Eq. (8.3.4) can be used for other tube diameters if the following correction factor is applied:

\[
q'^*_{\text{crit}} = q'^*_{\text{crit, kmm}} \left( \frac{8}{D} \right)^{0.5} \tag{8.3.6}
\]

where \( D \) is the tube diameter in \([\text{mm}]\) and \( q'^*_{\text{crit, kmm}} \) is the critical heat flux obtained from Eq. (8.3.4).

In a similar manner, the critical quality given by Eq. (8.3.5) can be used for other tube diameters with the following correction factor:

\[
x_{\text{crit}} = x_{\text{crit, kmm}} \left( \frac{8}{D} \right)^{0.15} \tag{8.3.7}
\]
Here $x_{\text{crt}} |_{8 \text{mm}}$ is the critical quality obtained from Eq. (8.3.5) and $D$ is the tube diameter in [mm].

### 8.4 Post-CHF Heat Transfer

As indicated in Fig. 8.0.1, if the flow boiling process exceeds the dryout condition, the post-dryout heat transfer will occur. This situation is valid for high-quality flow boiling crisis and is termed as the post-dryout heat transfer or the **mist flow evaporation**.

At low qualities and subcooled conditions the DNB may occur and will be followed by the **convective film boiling** at the heated walls. The film boiling is associated with sudden and very high wall temperature increase. A sustained film boiling at high mass flux levels causes wall temperatures so high that most conventional materials would melt. Due to that the DNB and the following film boiling must be avoided in fuel assemblies of LWRs. In addition, even experimental measurements of DNB and film boiling are quite difficult.

**Convective film boiling**

For convective film boiling at low to moderate rates the flow takes on the so-called inverted annular pattern. The vapor film is covering the heated wall and isolating water from the wall. The film is generally not smooth but exhibits irregularity at random locations. Several investigators have linked these distortions of the water-vapor interface to the growth of waves due to interface instability.

For laminar conduction-dominant transport, the heat transfer coefficient through the vapor film is given by

$$h = \frac{\lambda_g}{\delta}.$$  \hspace{1cm} (8.4.1)

Here $\delta$ is the vapor film thickness and $\lambda_g$ is the vapor conductivity. The vapor film thickness can be obtained from a simple one-dimensional model. Substituting the predicted film thickness to (8.4.1) yields,

$$h = \left[ \frac{\rho_g \left( \rho_l - \rho_g \right) g H_{fg} \lambda_{fg}^2}{4 (z - z_{\text{CHF}}) \mu_g (T_C - T_{\text{sat}})} \right]^{1/4}.$$  \hspace{1cm} (8.4.2)

This simplified model, however, takes no account for the bubbles that rise with film and its accuracy is not very good.

Another model based on Bromley’s equation for film boiling from horizontal cylinder and using the Helmholtz length scale is as follows,
\[ h = 0.62 \left[ \frac{\hat{\lambda}_g \rho_g (\rho_f - \rho_g) g H_{fg}}{\mu_g (T_C - T_{sat}) \hat{\lambda}_H} \right]^{1/4}, \]  

(8.4.3)

where Helmholtz-unstable wavelength is defined as,

\[ \hat{\lambda}_H = 16.24 \left[ \frac{\sigma^4 H_{fg}^3 \mu_g^2}{\rho_g (\rho_f - \rho_g) g^5 \hat{\lambda}_g^3 (T_C - T_{sat})^2} \right]^{1/2}, \]  

(8.4.4)

**Mist flow evaporation**

For mist flow evaporation several different heat transfer mechanisms may play role:

1. Convective heat transfer from the wall to the vapor
2. Convective heat transfer from the vapor to the entrained droplets
3. Evaporation of droplets that collide with the wall and wet its surface
4. Evaporation of droplets that come to close proximity to the wall but do not wet the surface
5. Radiation heat transfer from the wall to the droplets
6. Radiation heat transfer from the wall to the vapor
7. Radiation heat transfer from vapor to droplets

Each of these mechanisms can be evaluated separately leading to a quite complex model. The heat exchange mechanisms in the mist-flow evaporation heat transfer are shown in Fig. 8.4.1.

![Figure 8.4.1. Heat exchange mechanisms in the mist-flow evaporation heat transfer.](image)

Another, simplified approach is to correlate the heat transfer coefficient based on experimental data. Based on best fit to data, Groeneveld proposed the following correlation for heat transfer in the dispersed flow regime:
\[ \text{Nu}_g = \frac{hD}{\lambda_g} = a \left[ \frac{GD}{\mu_g} \left( x + \frac{\rho_g}{\rho_f} (1 - x) \right) \right]^b \text{Pr}_{g,w}^c Y^d, \]  

(8.4.5)

where

\[ Y = 1 - 0.1 \left( \frac{\rho_f}{\rho_g} - 1 \right)^{0.4} (1 - x)^{0.4}. \]  

(8.4.6)

Note that $\text{Pr}_{g,w}$ is the vapor Prandtl number evaluated at the wall temperature and all other parameters are evaluated at saturation temperature. Values of the constants $a$, $b$, $c$ and $d$ are provided for tubes and annuli only, see Table 8.4.1.

**Table 8.4.1 Coefficients in the Groeneveld’s correlation (8.4.4)**

<table>
<thead>
<tr>
<th>Geometry</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tubes</td>
<td>0.00109</td>
<td>0.989</td>
<td>1.41</td>
<td>-1.15</td>
</tr>
<tr>
<td>Annuli</td>
<td>0.0520</td>
<td>0.688</td>
<td>1.26</td>
<td>-1.06</td>
</tr>
<tr>
<td>Tubes and annuli</td>
<td>0.00327</td>
<td>0.901</td>
<td>1.32</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

The validity of the correlation is limited to the following ranges: $2.5 < D < 25$ mm, $68 < p < 215$ bar, $700 < G < 5300$ [kg m$^{-2}$ s$^{-1}$], $0.1 < x < 0.9$, $120 < q'' < 2100$ kW m$^{-2}$, $0.88 < \text{Pr}_{g,w} < 2.21$, $0.706 < Y < 0.976$ and $6.6 \times 10^4 < \left( \frac{GD}{\mu_g} \right)^{0.4} x + \frac{\rho_g}{\rho_f} (1 - x) < 1.3 \times 10^6$.

### 8.5 Void Fraction Distribution in Boiling Channels

When both the vapor and the liquid phase of coolant are flowing in a channel the flow is termed as two-phase flow. The fundamental feature that distinguishes two-phase flows from single-phase flows is the presence of an interface between the two phases. The shape and the motion of the interface is usually a part of the solution and is not known \( à \) priori. That makes a solution of two-phase flow problems much more difficult than the solution of single phase problems.

The objective of this lecture is to present the basic features of two-phase flows in fuel assemblies of nuclear reactors. It will be shown how the basic parameters, like void fraction and pressure drop can be predicted. It will be assumed that two-phase mixture consists of a gas (or vapor) phase, always designed with the $g$ subscript, and the liquid phase, designed with the $l$ subscript.

**Static quality** of the two-phase mixture is defined as a ratio of the vapor mass to the total mixture mass,

\[ \chi = \frac{M_g}{M_g + M_l} = \frac{M_g}{M}. \]  

(8.5.1)
The volume of the vapor in the mixture divided by the total volume of the mixture is termed the **void fraction**. In analogy to Eq. (8.5.1), the volume-averaged void fraction is computed as,

$$\alpha = \frac{V_g}{V_g + V_l} = \frac{V_g}{V}.$$  \hspace{1cm} (8.5.2)

The static quality can be expressed in terms of the volume-averaged void fraction (and vice-versa) using the following relationships: \(M_g = \rho_g V_g\) and \(M_l = \rho_l V_l\),

$$\chi = \frac{\rho_g V_g}{\rho_g V_g + \rho_l V_l} = \frac{1}{1 + \frac{\rho_l V_l}{\rho_g V_g}} = \frac{1}{1 + \frac{\rho_l (1 - \alpha)}{\rho_g (\frac{\chi}{\alpha})}},$$  \hspace{1cm} (8.5.3)

and

$$\alpha = \frac{M_g / \rho_g}{M_g / \rho_g + M_l / \rho_l} = \frac{1}{1 + \frac{\rho_g M_l}{\rho_l M_g}} = \frac{1}{1 + \frac{\rho_g (1 - \chi)}{\rho_l (\chi)}}.$$  \hspace{1cm} (8.5.4)

For flowing mixtures, these quantities can be computed in terms of the cross-sectional areas in the channel occupied by liquid, \(A_l\), and vapor, \(A_g\). The cross-section averaged void fraction is thus given as,

$$\alpha = \frac{A_g}{A_g + A_l}.$$  \hspace{1cm} (8.5.5)

The mass flow rate is usually represented by symbol \(W\) and has unit [kg/s]. The individual mass flow rates of liquid and vapor will be \(W_l\) and \(W_g\), respectively, and their sum will be equal to \(W\). The mass flow rates of vapor and liquid can be expressed in terms of cross-section areas and phase velocities, \(u_l\) and \(u_g\), as follows,

$$W_l = \rho_l u_l A_l, \hspace{1cm} W_g = \rho_g u_g A_g.$$  \hspace{1cm} (8.5.6)

In boiling channels it is convenient to use the fraction of the total mass flow which is composed of vapor and liquid. The **mass flow quality** is defined as,

$$x = \frac{W_g}{W_g + W_l} = \frac{W_g}{W}.$$  \hspace{1cm} (8.5.7)
In a similar manner as for the static quality, the flowing quality can be expressed in terms of the cross-section averaged void fraction as,

\[ x = \frac{\rho_g u_g A_g}{\rho_g u_g A_g + \rho_l u_l A_l} = \frac{1}{1 + \frac{\rho_l u_l A_l}{\rho_g u_g A_g}} = \frac{1}{1 + \frac{\rho_l}{\rho_g} \cdot \frac{u_l}{u_g} \left( \frac{1 - \alpha}{\alpha} \right)}. \quad (8.5.8) \]

The void fraction becomes,

\[ \alpha = \frac{W_g / (\rho_g u_g)}{W_g / (\rho_g u_g) + W_l / (\rho_l u_l)} = \frac{1}{1 + \frac{W_l}{W_g} \cdot \frac{\rho_g}{\rho_l} \cdot \frac{u_g}{u_l} \left( \frac{1 - \alpha}{\alpha} \right)}. \quad (8.5.9) \]

### 8.5.1 Homogeneous Equilibrium Model

In this model the phases are assumed to be in equilibrium – both in the mechanical and the thermodynamic sense. The mechanical equilibrium assumption means that both phases are well mixed and move with the same velocity. The thermodynamic equilibrium assumption means that both phases co-exist in the saturation form; none of them is in the superheated or subcooled condition.

The energy balance for a two-phase mixture can be written as,

\[ W \cdot H_c(z) + q'' \cdot P_b \cdot dz = W \cdot [H_c(z) + dH_c] \quad (8.5.1-1) \]

which is very similar to the energy balance for the single-phase flow.

The enthalpy distribution is thus obtained as,

\[ W \cdot H_c(z) + q'' \cdot P_b \cdot dz = W \cdot [H_c(z) + dH_c] \Rightarrow \frac{dH_c(z)}{dz} = \frac{q'' \cdot P_b}{W} \quad (8.5.1-2) \]

For constant heat flux this yields,

\[ H_c(z) = H_{ci} + \frac{q'' \cdot P_b}{W} z \quad (8.5.1-3) \]

Eq. (8.5.7) defines so called mass flow quality. In addition, there is a definition of the mixture quality based on enthalpies,

\[ x = \frac{H - H_f}{H_{fg}} \quad (8.5.1-4) \]
For HEM the flow and the thermodynamic quality are equivalent, that is $x_H = x$.
Combining Eq. (8.5.1-3) and Eq. (8.5.1-4) yields,

$$x(z) = x_i + \frac{q^* \cdot P}{W \cdot H_{fg}} z$$  \hspace{1cm} (8.5.1-5)

The void fraction has been expressed as a function of local flow quality in Eq. (8.5.9).
Since in the Homogeneous Equilibrium Model the phasic velocities are equal, e.g.,

$$U_g = U_l,$$  \hspace{1cm} (8.5.1-6)

The expression for the void fraction reduces to,

$$\alpha = \frac{1}{1 + \rho_g \cdot \left(\frac{1-x}{x}\right)}.$$  \hspace{1cm} (8.5.1-7)

Combining Eq. (8.5.1-7) with (8.5.1-5) gives the distribution of the void fraction as a function of the axial distance in a channel.

8.5.2 Drift Flux Model

Presence of the vapor phase has a significant influence on the local value of the mean coolant density. This, in turn, influences the local neutron flux and thus the local power. Clearly, there is a feedback between the local power and the local void fraction. Due to that it is very important to be able to accurately estimate the local value of the void fraction.

Eqs (8.5.4) and (8.5.9) give expressions for the void fraction in stationary and flowing mixture, respectively. In case of the stationary mixture Eq. (8.5.4) gives an exact expression for the void fraction. Knowing the densities of phases and the static quality, the corresponding void fraction can be computed.

**Example 8.5.2-1:** Comparison between the static quality and the void fraction.
A tank contains a mixture of air and water, with $M_a = 1$ kg and $M_w = 999$ kg. Calculate the void fraction of the mixture assuming $\rho_a = 1$ kg/m$^3$ and $\rho_w = 1000$ kg/m$^3$.

**Solution:**
The static quality is found as $\chi = 1/(1 + 999) = 0.001$ and the void fraction as $\alpha = 1/(1 + (1 \cdot 0.999)/(1000 \cdot 0.001)) = 0.50025$. Thus, it would seem that the mass fraction of air is negligible (only 0.1%), whereas the air fills more then half of the vessel (50.025%). This example illustrates the importance of considering both the void fraction and the quality.
More interesting is the case with the flowing mixture, unfortunately, an accurate prediction of void fraction is much more complex. Apparently Eq. (8.5.9) can be used for that purpose; however, one has to know the slip ratio, \( S = u_g / u_l \). This parameter is not constant and depends in a complex manner on flow quality \( x \), pressure, mass flux and flow regime. Typically the slip ratio has been found from two-phase flow measurements.

In the simplest case it can be assumed that the phases move with the same velocity and thus the slip ratio is equal to 1. This assumption is a fundamental part of the so-called Homogeneous Equilibrium Model (HEM). According to this model, void fraction can be found from Eq. (8.5.4) and is a function of local quality and pressure only.

A direct and essential extension of the slip ratio approach is the Drift Flux Model (DFM) proposed by Zuber and Findlay (1965). The model enables the slip between the phases as well as it takes into account the spatial pattern of the flow.

Given a two-phase mixture flowing in a pipe, the following local quantities can be defined:

\[
\begin{align*}
j_i & = (1 - \alpha) u_i , \\
j_g & = \alpha u_g , \\
j & = j_i + j_g , \\
u_{gi} & = u_g - j = (1 - \alpha) (u_g - u_i) ,
\end{align*}
\]

where \( \alpha \) is the local void fraction and \( u_k \) is the local velocity of phase \( k \) (\( k = g \) for the vapor phase and \( k = l \) for the liquid phase). From Eq. (8.5.2-4) one gets,

\[
\alpha u_{gi} = \alpha u_g - \alpha j = j_g - \alpha j .
\]

Eq. (8.5.2-5) can be integrated over the channel cross-section to obtain,

\[
\int_A \alpha u_{gi} dA = \int_A j_g dA - \int_A \alpha j dA ,
\]

where \( A \) is the channel cross-section area. The following definitions can be introduced:

\[
U_{gi} \equiv \frac{\int_A \alpha u_{gi} dA}{\int_A \alpha dA} = \frac{\int_A \alpha u_{gi} dA}{\langle \alpha \rangle A} ,
\]

\[
\int_A \alpha j dA \equiv C_0 \int_A \alpha dA \cdot \int_A j dA = C_0 \cdot \langle \alpha \rangle \cdot J \cdot A ,
\]

\[
J_g = \frac{1}{A} \int_A j_g dA .
\]
Combining Eqs (8.5.2-6) through (8.5.2-9) yields,

\[ \langle \alpha \rangle = \frac{J_g}{C_0 J + U_{gi}} \]  \hspace{1cm} (8.5.2-10)

Eq. (8.5.2-10) is the celebrated Drift-Flux void correlation. It expresses the cross-section mean void fraction \( \langle \alpha \rangle \) in terms of channel mean superficial velocity of gas, \( J_g \), total superficial velocity, \( J \), and two parameters, \( C_0 \) and \( U_{gi} \). The first parameter is the so-called \textit{drift-flux distribution parameter}. Its physical interpretation is clear from Eq. (8.5.2-8) and is simply a covariance coefficient for cross-section distributions of void fraction and total superficial velocity. The second coefficient is the so-called \textit{drift velocity} and can be interpreted as cross-section averaged difference between gas velocity and superficial velocity, using local void fraction as a weighting function.

The drift-flux parameters are not constant and depend on flow conditions. For high pressure steam-water flows the parameters are independent on two-phase flow pattern and are well approximated with the following expressions,

\[ C_0 = 1.13 \]  \hspace{1cm} (8.5.2-11)

\[ U_{gi} = 1.41 \left( \frac{\sigma_g (\rho_l - \rho_g)}{\rho_l^2} \right)^{0.25} \]  \hspace{1cm} (8.5.2-12)

Table 8.5.2-1 gives expressions for drift-flux parameters, which are valid in a wide range of flow conditions.

<table>
<thead>
<tr>
<th>Flow pattern</th>
<th>Distribution parameter</th>
<th>Drift velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubbly ( 0 &lt; \alpha \leq 0.25 )</td>
<td>[ C_0 = \begin{cases} 1 - 0.5p/p_c &amp; p/p_c &lt; 0.5 \ 1.2 &amp; p/p_c \geq 0.5 \ 1.4 - 0.4p/p_c &amp; \end{cases} ] \quad \begin{cases} D \geq 0.05m \ D &lt; 0.05m \end{cases}</td>
<td>[ U_{gi} = 1.41 \left( \frac{\sigma_g (\rho_l - \rho_g)}{\rho_l^2} \right)^{0.25} ]</td>
</tr>
<tr>
<td>Slug/churn ( 0.25 &lt; \alpha \leq 0.75 )</td>
<td>( C_0 = 1.15 )</td>
<td>( U_{gi} = 0.35 \left( \frac{gD(\rho_l - \rho_g)}{\rho_l} \right)^{0.5} )</td>
</tr>
<tr>
<td>Annular ( 0.75 &lt; \alpha \leq 0.95 )</td>
<td>( C_0 = 1.05 )</td>
<td>( U_{gi} = 23 \left( \frac{\mu_l j_l}{\rho_g D} \right)^{0.5} \left( \frac{\rho_l - \rho_g}{\rho_l} \right) )</td>
</tr>
</tbody>
</table>
The drift-flux void correlation given by Eq. (8.5.2-10) can be expressed in terms of the quality and mass flux as follows,

\[
\langle \alpha \rangle = \frac{J_g}{C_0 J + U_{gj}} = \frac{1}{C_0 \left( \frac{J_g}{J_g + J_l} + \frac{U_{gj}}{J_g} \right)} = \frac{1}{C_0 \left( 1 + \frac{\rho_g G_l}{G_g} \right)} + \frac{\rho_g U_{gj}}{G_g}. \tag{8.5.2-13}
\]

It can be noticed that using \( C_0 = 1 \) and \( U_{gj} = 0 \) gives the homogeneous void correlation. Another observation is that for any \( C_0 \geq 1 \) and \( U_{gj} > 0 \), void fraction predicted from Eq. (8.5.2-13) is lower than void fraction predicted from the homogeneous model.

**Example 8.5.2-2:** Find expression for the slip ratio \( S \) that will result in a void correlation which is equivalent to the drift-flux correlation given by Eq. (8.5.2-13).

**Solution:**

\[
S = C_0 + (C_0 - 1) \frac{\rho_l}{C_0 \rho_g (1-x) + U_{gj} \rho_l} \tag{8.5.2-14}
\]

**8.6 Pressure Distribution in Boiling Channels**

**Momentum balance in a boiling channel:**
The pressure drop in an arbitrary channel can be found from a force-momentum balance in the axial direction (forces are projected onto channel axis). A differential portion of the channel is shown in Fig. 8.6.1.
The force-momentum balance for phase $k$ ($k = l$ for liquid and $k = g$ for gas) is as follows,

**Pressure force balance:**

$$p_k A_k - (p_k + dp_k)(A_k + dA_k) \Rightarrow -p_k dA_k - A_k dp_k,$$

**Wall friction force:**

$$-dF_{kw},$$

**Interfacial friction force:**

$$-dF_{ki},$$

**Gravity force:**

$$-A_k dz \rho_k g \sin \varphi,$$

**Momentum change:**

$$-(W_k + dW_k)(U_k + dU_k) + W_k U_k \rightarrow -dW_k U_k - W_k dU_k$$

**Momentum change due to phase change:**

$$\sum_{j \neq k} dA_{i,kj} \Gamma_{kj} U_{i,kj}$$

All terms have to sum to zero to preserve the linear momentum,

$$- A_k dp_k - p_k dA_k - dF_{kw} - dF_{ki} - A_k \rho_k g \sin \varphi dz - d(W_k U_k) +$$

$$\sum_{j \neq k} dA_{i,kj} \Gamma_{kj} U_{i,kj} = 0 \quad \text{(8.6.1)}$$

Here $A_{i,kj}$ is the interfacial area between phases $j$ and $k$, $\Gamma_{kj}$ is evaporation rate from phase $j$ to phase $k$ and $U_{i,kj}$ is the velocity of phase $j$ at the interface with phase $k$. 
Dividing Eq. (8.6.1) by $Adz$ gives the following differential momentum equation,

$$-\alpha_k \frac{dp_k}{dz} = \frac{p_k}{A} \frac{d(\alpha_k A)}{dz} + \frac{dF_{kw}}{Adz} + \frac{dF_{ki}}{Adz} + \alpha_k \rho_k g \sin \varphi +$$

$$\frac{1}{A} \frac{d}{dz} \left( \frac{G_k^2}{\rho_k \alpha_k} \right) - \sum_{j \neq k} a_{i,kj} \Gamma_{kj} U_{i,kj},$$

(8.6.2)

where $a_{i,kj} = A_{i,kj} / (Adz)$ is the interfacial area density, $\alpha_k = A_k / A$ is void fraction and $G_k = W_k / A_k$ is the mass flux for phase $k$.

Equation (8.6.2) shows all terms which contribute to the total pressure drop in phase $k$. The first term on the right-hand-side represents the shape (or form) pressure drop due to the area change, the second term represents the friction (or skin) pressure drop, the third term is the interfacial friction pressure drop, the forth term is the gravity pressure drop, the fifth term is the acceleration pressure drop and the sixth term is the phase-change pressure drop.

Equation (8.6.2) is rarely used in practical applications. Most often the total pressure drop, mean over all phases, is calculated. Adding Eqs. (8.6.2) for all phases $k$, and combining the pressure terms, the following relation for overall pressure drop is obtained,

$$-\frac{dp}{dz} = \frac{p}{A} \frac{dA}{dz} + \sum_k \frac{dF_{kw}}{Adz} + \sum_k \frac{dF_{ki}}{Adz} + g \sin \varphi \sum_k \alpha_k \rho_k +$$

$$\frac{1}{A} \frac{d}{dz} \left( \sum_k A \frac{G_k^2}{\rho_k \alpha_k} \right) - \sum_k \sum_{j \neq k} a_{i,kj} \Gamma_{kj} U_{i,kj},$$

(8.6.3)

The third and the last term on the right-hand-side are equal to zero due to force balance and velocity continuity at the interface, respectively. Finally the momentum balance relation can be written as,

$$-\frac{dp}{dz} = \frac{p}{A} \frac{dA}{dz} + \left( \frac{dp}{dz} \right)_w + \rho_m g \sin \varphi + \frac{1}{A} \frac{d}{dz} \left( \frac{G^2 A}{\rho_M} \right).$$

(8.6.4)

Here two definitions for the mixture density have been introduced.

**Homogeneous (static) mixture density:**

$$\rho_m = \sum_k \rho_k \alpha_k$$

**Dynamic mixture density:**
\[
\rho_M = \left( \sum_k \frac{x_k^2}{\rho_k \alpha_k} \right)^{-1}
\]

Equation (8.6.4) describes in a general form the pressure gradient change in a boiling channel of arbitrary shape. There are four distinct terms on the right-hand-side of Eq. (8.6.4) Based on the previous classification of various terms appearing in Eq. (8.6.2) the terms can be identified and explained as follows. The first term is the shape (or form) pressure drop term, and represents the pressure gradient change due to the change of the channel cross-section area \(A\). The second term is the friction pressure loss term, and represents the pressure gradient change due to the friction between two-phase mixture and the channel walls. The third term represents the pressure gradient change due to the gravity and is called, as could be expected, the gravity term. Finally, the fourth and the last term is the acceleration term, since it represents the pressure gradient change due to the mixture acceleration in the channel. All these terms will be described below in more detail.

**Shape pressure drop:**

The shape pressure drop term can be evaluated as,

\[- \left( \frac{dp}{dz} \right)_{shp} = \frac{p}{A} \frac{dA}{dz} \]

provided the channel cross-section area \(A\) is a known function of the axial coordinate \(z\). For channels with constant cross-section area the term is equal to zero.

**Friction pressure loss:**

The total friction between walls and all phases has been defined as,

\[- \left( \frac{dp}{dz} \right)_w = \frac{1}{A} \sum_k \frac{dF_{w,k}}{dz} \]

(8.6.5)

Using Eq. (7.4.8) in Section 7.4, the friction force between phase \(k\) and wall can be expressed as,

\[
\frac{dF_{w,k}}{dz} = \frac{dz \cdot P_{w,k} \cdot \tau_{w,k}}{dz} = \frac{C_{f,k}}{2} \rho_k U_k^2 = P_{w,k} \frac{C_{f,k}}{2 \rho_k} G_k^2.
\]

(8.6.6)

Here \(P_{w,k}\) is the channel perimeter which is in contact with phase \(k\). Combining Eqs. (8.6.5) and (8.6.6) yields,

\[- \left( \frac{dp}{dz} \right)_w = \frac{1}{A} \sum_k P_{w,k} \frac{C_{f,k}}{2 \rho_k} G_k^2.
\]

(8.6.7)
Equation (8.6.7) is not practical to predict the wall-friction pressure drop unless \( P_{w,k} \) is known. In the majority of two-phase flow situations this is the case. Instead, the two-phase friction is expressed in terms of so-called two-phase friction multipliers, which are defined as,

\[
\phi_s^2 = \frac{(dp/dz)_{w,wp}}{(dp/dz)_{w,l}}, \tag{8.6.8}
\]

\[
\phi_{lo}^2 = \frac{(dp/dz)_{w,wp}}{(dp/dz)_{w,lo}}, \tag{8.6.9}
\]

\[
\phi_g^2 = \frac{(dp/dz)_{w,wp}}{(dp/dz)_{w,g}}, \tag{8.6.10}
\]

where pressure drop with index \( w,l \) denotes the frictional pressure gradient that would result if the liquid flowed alone through the channel at a mass flow rate equal to \( G_l \), \( w,lo \) denotes the frictional pressure gradient that would result if the liquid flowed alone through the channel at a mass flow rate equal to \( G \) and \( w,g \) denotes the frictional pressure gradient that would result if the vapor flowed alone through the channel at a mass flow rate equal to \( G_g \).

Using Eq. (8.6.9) the two-phase pressure gradient due to friction can be found as,

\[
\left( \frac{dp}{dz} \right)_{w,wp} = \phi_{lo}^2 \left( \frac{dp}{dz} \right)_{w,lo}, \tag{8.6.11}
\]

where the fictitious single-phase (liquid-only) pressure gradient is found from the standard expression (see Eq. 7.4.12),

\[
-\left( \frac{dp}{dz} \right)_{w,lo} = \frac{2C_{f,lo}}{\rho_lD} G^2. \tag{8.6.12}
\]

Here the friction coefficient \( C_{f,lo} \) can be found from any of expressions (6.3.26) through (6.3.30), depending on flow conditions. The two-phase friction multiplier \( \phi_{lo}^2 \) has to be found from any expressions given below.

The two-phase flow can be treated as an equivalent single-phase flow, applying the so-called homogeneous model of momentum transport. In such case, the two-phase pressure gradient can be determined as,
\[- \left( \frac{dp}{dz} \right)_{w, tp} = \frac{2C_{f, tp} G^2}{\rho_m D}. \]  

(8.6.13)

In this expression \( C_{f, tp} \) is an effective Fanning friction factor for the two-phase flow and \( \rho_m \) is the mixture density. Combining Eqs. (8.6.11) through (8.6.13) yields,

\[ \phi_{lo}^2 = \frac{C_{f, tp}}{C_{f, tp} \rho_l} \frac{\rho_l}{\rho_m} = \frac{C_{f, tp}}{C_{f, tp} \rho_l} \left[ 1 + \left( \frac{\rho_l}{\rho_g} - 1 \right) x \right]. \]  

(8.6.14)

The friction factors can usually be expressed as functions of the Reynolds number. Using Eq. (6.3.28) as a prototype of such a function, the friction factors read as follows,

\[ C_{f, lo} = A \cdot Re_{lo}^{-a} \quad C_{f, tp} = B \cdot Re_{tp}^{-b} \]  

(8.6.15)

Assuming next that coefficients in Eq. (8.6.15) for single-phase and two-phase are equal, that is \( A = B \) and \( a = b \), Eq. (8.6.14) becomes,

\[ \phi_{lo}^2 = \left( \frac{\mu_m}{\mu_i} \right)^b \left[ 1 + \left( \frac{\rho_l}{\rho_g} - 1 \right) x \right]. \]  

(8.6.16)

The mixture viscosity appearing in Eqs. (8.6.15) and (8.6.16) can be defined in different ways. It is customary to use some kind of weighted mean value, employing quality as the weighting factor. Three examples of the mixture viscosity are as follows,

\[ \frac{1}{\mu_m} = \frac{x}{\mu_g} + \frac{1-x}{\mu_i}, \]  

(8.6.17)

\[ \mu_m = x \mu_g + (1-x) \mu_i, \]  

(8.6.18)

\[ \frac{\mu_m}{\rho_m} = \frac{x \mu_g}{\rho_g} + \frac{(1-x) \mu_i}{\rho_i}. \]  

(8.6.19)

Using Eq. (8.6.17) in (8.6.16), and taking \( a = b = 0.25 \) (see the Blasius correlation) yields,

\[ \phi_{lo}^2 = \left[ 1 + \left( \frac{\mu_i}{\mu_g} - 1 \right) x \right]^{-0.25} \left[ 1 + \left( \frac{\rho_l}{\rho_g} - 1 \right) x \right]. \]  

(8.6.20)

Combining Eqs. (8.6.11), (8.6.12) and (8.6.20) yield the following expression for the friction pressure term:
The above equation is one example of many possible formulations, depending on the choice of the two-phase flow multiplier.

Two phase flow multipliers play an important role in the prediction of the pressure drop in two-phase flows. As can be expected, there are plenty of correlations available in the open literature. In addition, manufacturers of the nuclear fuel perform pressure drop tests using their own designs and develop fuel-specific two-phase flow multipliers. As an example, a model proposed by Lockhart and Martinelli will be described below.

Lockhart and Martinelli proposed a generalized correlation method for determining the two-phase multiplier $\phi^2_t$ and $\phi^2_g$ from which the frictional pressure gradient can be predicted for adiabatic gas-liquid flow in round tubes. The correlations are based on data from a series of measurements of adiabatic two-phase flow in horizontal tubes. The correlations are as follows,

$$\phi^2_t = 1 + \frac{C}{X} + \frac{1}{X^2}, \quad (8.6.21)$$

$$\phi^2_g = 1 + C X + X^2. \quad (8.6.22)$$

Here $X$ is the Martinelli parameter, defined as

$$X = \left[ \frac{(dp/dz)_l}{(dp/dz)_g} \right]^{0.5} \quad (8.6.23)$$

This parameter will have different forms depending on whether phases are laminar or turbulent. As an example assume that both phases are turbulent, then,

$$-\frac{dp}{dz}_l = A \left( \frac{G_l D}{\mu_l} \right)^{-a} \frac{G^2_l}{2 \rho_l}; \quad -\frac{dp}{dz}_g = A \left( \frac{G_g D}{\mu_g} \right)^{-a} \frac{G^2_g}{2 \rho_g}$$

and the $X$-parameter in Eq. (8.6.23) becomes,
The \( tt \) subscript indicates that both phases are turbulent. Other possible subscript combinations will be \( lt \), \( tl \) and \( ll \), meaning laminar liquid – turbulent vapor, turbulent liquid – laminar vapor and both phases laminar, respectively.

The recommended value of the constant \( C \) in Eqs (8.6.21-22) differs, depending on the flow regime associated with the flow of the vapor and the liquid alone in the channel. There are four possible combinations in which each phase can be either laminar or turbulent. The values of \( C \) are shown in Table 8.6.1.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Gas -&gt;</th>
<th>Laminar</th>
<th>Turbulent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar</td>
<td>5</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Turbulent</td>
<td>10</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

The two phase flow multiplier \( \phi_t^2 \) can be expressed in terms of \( \phi_l^2 \) as follows,

\[
\phi_t^2 = \left( \frac{dp/dz}_{w,lp} \right) = (1 - x)^{1.75} \phi_l^2.
\]

**Gravity pressure drop:**
The gravity pressure drop term is defined as,

\[
- \left( \frac{dp}{dz} \right)_{grav} = \rho_m g \sin \varphi.
\]

Here \( \sin \varphi = 1 \) for vertical channels and \( \sin \varphi = 0 \) for horizontal channels. Since, in general, the mixture density \( \rho_m \) can change along channel, the pressure gradient will change accordingly.

**Acceleration pressure drop:**
The fourth term on the right-hand-side of Eq. (8.6.4) is termed as the acceleration pressure drop and can be evaluated as:

\[
- \left( \frac{dp}{dz} \right)_{acc} = \frac{1}{A} \frac{d}{dz} \left( \frac{G^2 A}{\rho_M} \right)
\]

For constant \( G \) and \( A \), this expression reduces to:
\[- \left( \frac{dp}{dz} \right) \bigg|_{acc} = G^2 \frac{dz}{dx} \left( \frac{1}{\rho_M} \right) \]

Substituting the expression for the dynamic mixture density \( \rho_M \) into the above equation gives,

\[- \left( \frac{dp}{dz} \right) \bigg|_{acc} = G^2 \frac{dz}{dx} \left[ \frac{x^2}{\rho_g} + \frac{(1-x)^2}{(1-\alpha)\rho_l} \right] \]

Equation (8.6.4) describes all pressure drop terms that appear in an arbitrary channel with two-phase flow which is smooth enough that there are no local pressure drops. If the channel contain local obstacles or sudden cross-section area changes, the corresponding pressure drops must be added to the previously described ones. This procedure is very much the same as that described in Chapter 7 for single-phase flows. However, some important differences, described below, have to be taken into account while applying the single-phase theory to two-phase flows.

**Local pressure drop:**

Local pressure losses for single phase have been discussed in Chapter 7. The single-phase flow theory will be now extended to two-phase flow situations. Using sections “1” and “2” shown in Fig. 7.4.1, the momentum balance equation can be written

\[ F = (1-\alpha_1) \rho_1 U_{11} A_{11} U_{11} + \alpha_1 \rho_g U_{g1} A_{g1} U_{g1} - \]

\[ [(1-\alpha_2) \rho_1 U_{12} A_{12} U_{12} + \alpha_2 \rho_g U_{g2} A_{g2} U_{g2} + p_1 A_1 - p_2 A_2 \]  

(8.6.24)

With similar arguments as for the single-phase flow case, the force \( F \) is taken equal to \( p_1(A_2-A_1) \), and using \( s = A_2/A_1 \), Eq. (8.6.24) yields

\[ p_2 - p_1 = \frac{G^2}{\rho_1} \left[ \frac{(1-x)^2}{1-\alpha_1} + \frac{\rho_l}{\rho_g} \frac{x^2}{\alpha_1} \right] - s \left[ \frac{(1-x)^2}{1-\alpha_2} + \frac{\rho_l}{\rho_g} \frac{x^2}{\alpha_2} \right] \]

(8.6.25)

The corresponding irreversible pressure drop, assuming incompressible two-phase flow with \( \alpha_1 = \alpha_2 = \alpha \), becomes

\[ - \Delta p_l = \frac{G^2}{\rho_l} (1-s) \left[ \left( \frac{1-x}{1-\alpha} + \frac{\rho_l}{\rho_g} \frac{x}{\alpha} \right) - \frac{1}{2} \left[ \frac{(1-x)^3}{(1-\alpha)^2 \rho_l} \frac{\rho_l}{\rho_g} \frac{x^2}{\alpha} \right] \right] \]

(8.6.26)

It has been assumed that there is no phase change between sections “1” and “2”, that is \( x_1 = x_2 \). However, void fractions at the two sections are usually not equal to each other. It
has been observed that void fraction significantly increases downstream of the enlargement. This effect disappears at some distance, however. This distance is evaluated to be between $L/D = 10$ up to 70. Nevertheless, Eq. (8.6.26) can be simplified by assuming the homogeneous model and the resultant irreversible pressure drop will become as follows,

$$-\Delta p_l = \left[ 1 + x \left( \frac{\rho_l}{\rho_g} - 1 \right) \right] \left( 1 - \frac{A_l}{A_2} \right)^2 \frac{G_i^2}{2 \rho_l}. \quad (8.6.27)$$

This equation can be compared with its equivalent for the single-phase flow through a sudden enlargement given by Eq. (7.4.19). As can be seen, a new term appears, which can be identified as a two-phase multiplier for the local pressure loss

$$\phi_{lo,d}^2 = \left[ 1 + x \left( \frac{\rho_l}{\rho_g} - 1 \right) \right]. \quad (8.6.28)$$

The subscript $lo,d$ is used to indicate that the multiplier given by Eq. (8.6.28) is valid for local losses, where the viscous effects can be neglected and only the density ratio between the two phases plays any role.

The corresponding irreversible pressure drop for homogeneous two-phase flow through a sudden contraction becomes (see Section 7.4 for single-phase equivalent)

$$-\Delta p_l = \left[ 1 + x \left( \frac{\rho_l}{\rho_g} - 1 \right) \right] \left( \frac{A_2}{A_c} - 1 \right)^2 \frac{G_2^2}{2 \rho_l}. \quad (8.6.29)$$

Even here the two-phase multiplier (8.6.28) appears together with the local loss coefficient valid for single-phase flows (see Table 7.4.1 for values of the local loss coefficient).

In general, a local irreversible pressure drop for two-phase flows can be expressed as

$$\Delta p_{l,\text{tp}} = \phi_{lo,d}^2 \Delta p_{l,lo}, \quad (8.6.30)$$

where $tp$ stands for two-phase and $lo$ for liquid only. As can be seen, the local pressure drop for two-phase flows can be obtained from a multiplication of the corresponding local pressure drop for single-phase flow and a proper two-phase multiplier. A similar principle is valid for the frictional pressure drop for single- and two-phase flows. It can be noted that the local two-phase multiplier given by Eq. (8.6.28) is not the same as the frictional two-phase multiplier, given by Eq. (8.6.20). The reason for this difference stems from the neglect of viscous losses in the case of the local pressure losses. Nevertheless, it should be mentioned that in many in practical calculations, the multiplier is assumed the same for both frictional and local pressure losses.
Integral pressure drop over a boiling channel

Equation (8.6.4) gives an expression for the pressure gradient change along a boiling channel. In practical calculation it is usually required to determine the over-all pressure drop in a channel of a given length and shape. The total pressure drop can be readily obtained from an integration of Eq. (8.6.4) along the channel length as follows,

\[-\int_0^L \frac{dp}{dz} dz = -[p(L) - p(0)] \equiv -\Delta p =\]

\[-\int_0^L \left( \frac{p}{A} \frac{dA}{dz} \right) dz + \int_0^L \left( \frac{dp}{dz} \right) dz + \int_0^L \rho_m g \sin \varphi dz + \int_0^L \frac{1}{A} \frac{d}{dz} \left( \frac{G^2 A}{\rho_{st}} \right) dz\]

Assuming that the channel has a constant cross-section area and using expressions for the friction, gravity and acceleration terms, the following expression is obtained,

\[-\Delta p = C_f \frac{2}{D} G^2 \int_0^L \phi_s dz + g \sin \varphi \int_0^L \left[ \rho_g \left( 1 - \alpha \right) \rho_l \right] dz + G^2 \int_0^L \frac{d}{dz} \left[ \frac{x^2 \rho_l}{\alpha \rho_g} + \frac{(1 - x)^2 \rho_l}{\left( 1 - \alpha \right) \rho_g} \right] dz\]

It is customary to introduce integral multipliers into the above equations which are defined as follows,

The acceleration multiplier:

\[r_2 \equiv \rho_l \int_0^L \frac{d}{dz} \left[ \frac{x^2 \rho_l}{\alpha \rho_g} + \frac{(1 - x)^2 \rho_l}{(1 - \alpha) \rho_g} \right] dz = \left[ \frac{x^2 \rho_l}{\alpha \rho_g} + \frac{(1 - x)^2 \rho_l}{(1 - \alpha) \rho_g} \right]_e - \left[ \frac{x^2 \rho_l}{\alpha \rho_g} + \frac{(1 - x)^2 \rho_l}{(1 - \alpha) \rho_g} \right]_i\]

Here subscripts \(e\) and \(i\) mean that the expression in the rectangular parentheses is evaluated at the channel exit \( (z=L) \) and at the channel inlet \( (z=0) \), respectively. For heated channel with \( x = \alpha = 0 \) at the inlet and \( x_e \) with \( \alpha_e \) at the outlet, the multiplier is as follows,

\[r_2 = \left[ \frac{x_e^2 \rho_l}{\alpha_e \rho_g} + \frac{(1 - x_e)^2 \rho_l}{(1 - \alpha_e) \rho_g} \right] - 1\]

The friction multiplier:

\[r_3 = \frac{1}{L} \int_0^L \phi_s dz\]

The gravity multiplier:

\[r_4 \equiv \frac{1}{L \rho_l} \int_0^L \left[ \rho_g \left( 1 - \alpha \right) \rho_l \right] dz = 1 - \left( \frac{\rho_l - \rho_g}{\rho_l} \right) \frac{1}{L} \int_0^L \alpha dz\]
The total channel pressure drop can be then found as,

\[-\Delta p = r_2 C_{f,lo} \frac{2L}{D} \frac{G^2}{\rho_l} + r_4 L \rho g \sin \varphi + r_2 \frac{G^2}{\rho_l} \]

If the channel contains a number of local losses \((i = 1, \ldots, N)\), the total pressure drop will be as follows,

\[-\Delta p = r_2 C_{f,lo} \frac{2L}{D} \frac{G^2}{\rho_l} + r_4 L \rho g \sin \varphi + r_2 \frac{G^2}{\rho_l} + \left( \sum_{i=1}^{N} \phi_{lo,i}^2 \xi_i \right) \frac{G^2}{2 \rho_l} \]

Here \(\xi_i\) indicates the local loss coefficient of the \(i\)-th obstacle and \(\phi_{lo,i}^2\) is the corresponding local two-phase flow multiplier.

**Exercises**

**Exercise 24:** Predict the location of the onset-of-nucleate-boiling point in a uniformly heated tube (8 mm internal diameter) with \(q'' = 0.5 \text{ MW m}^{-2}\) on the internal wall. The tube is cooled with water at 140 bar, inlet subcooling 70 K and mass flux 2000 kg m\(^{-2}\) s\(^{-1}\). Use Bowring’s model for the onset of nucleate boiling and Jens and Lottes’ correlation for subcooled boiling. Use saturated water properties at 140 bar pressure. What will be the difference in the location of the ONB point if Thom *et al.* model is used instead?

**Exercise 25:** For conditions as described in Exercise 24, find the location of the point where the fully-developed boiling starts.

**Exercise 26:** Subcooled water at \(p = 70\) bar flows into a vertical round tube with uniformly heated wall with \(q''=200 \text{ kW m}^{-2}\) on the internal wall surface. The inlet subcooling is 40 K, the mass flux is 1200 kg m\(^{-2}\) s\(^{-1}\) and the tube internal diameter is \(D = 10\) mm. Find the temperature distribution on the internal wall surface from the inlet to the point where the bulk water temperature is equal to the saturation temperature. Use saturated water properties at 70 bar pressure.

**Exercise 27:** Plot \(q''_c(z)\) using the Bowring correlation and assuming steam-water flow at 70 bar in a pipe with \(D = 10\) mm and length 3.5 m. The inlet subcooling is 10 K, total mass flux \(G = 1250 \text{ kg m}^{-2} \text{ s}^{-1}\), and the correlation parameters are given as \(F_{B1} = F_{B2} = F_{B3} = F_{B4} = 1\).

**Exercise 28:** Using the Levitan and Lantsman correlation for dryout (Eq. 8.3.5), plot \(x_{crit}\) as a function of pressure in a range \(9.8 < p < 166.6 \text{ bar}\) and using \(G\) as parameter with values \(G = 750, 1000, 1500\) and \(2000 \text{ [kg m}^{-2} \text{ s}^{-1}\]) for boiling upflow of water in a uniformly heated, vertical tube with 8 mm internal diameter. Give the value of the
pressure for which $x_{\text{crit}}$ becomes maximum. Use this pressure to plot $x_{\text{crit}}$ as a function of $G$.

**Exercise 29:** Calculate the heat transfer coefficient from the Groeneveld correlation (Eq. 8.4.5) for steam-water flow in a vertical tube: $D = 10$ mm, $G = 1200$ kg m$^{-2}$ s$^{-1}$, $p = 70$ bar, heat flux $q = 1.0$ MW m$^{-2}$ at axial positions where $x = 0.2, 0.4, 0.6$ and $0.8$. Assume that the vapor Prandtl number varies with the temperature as $\text{Pr}(T) = 1.05 - 8 * (T-623) * 10^{-4}$, where $T$ is in [K]. Check whether the validity ranges are satisfied.

**Exercise 30:** Plot void fraction in function of flow quality (in a range from 0 to 1) assuming flow of water and vapor mixture at saturation conditions and at pressure $p = 70$ bar. Compare two cases: in one case both phases have the same averaged velocity, in the other case the vapor phase flows with mean velocity which is 20% higher than the mean liquid velocity.

**Exercise 31:** Compare void fraction predicted from the homogeneous model and the drift-flux model for steam-water flow. Assume $p = 70$ bar, $G = 1200$ kg m$^{-2}$ s$^{-1}$ and Eqs (8.5.2-11) and (8.5.2-12) for drift-flux parameters. Plot void fraction in function of quality. Plot slip ratio given by Eq. (8.5.2-14) in function of quality for the same conditions.

**Exercise 32:** Plot two-phase multiplier (8.6.20) as a function of quality assuming two-phase flow of water-vapor mixture under 70 bar pressure.

**Exercise 33:** Plot two-phase multipliers (8.6.16) using three different definitions of mixture viscosity. Assume same conditions as in Exercise 32.

**Exercise 34:** Derive expressions for integral multipliers $r_2$, $r_3$ and $r_4$ for channels with saturated liquid at the inlet and with uniform heat flux distribution along channel length. Hint: in integrals along channel length use substitution: $dz = \text{const} * dx$, which is valid for uniformly heated channels.

**Exercise 35:** Solve Exercise 23 assuming constant heat flux applied to the channel and equal to 3 MW m$^{-2}$.

**Home Assignment # 4 (due to 05.02.16)**

Vertical smooth pipe with 8 mm internal diameter and length 3.6 m is uniformly heated with heat flux $q'' = 0.75$ MW/m$^2$. The pipe is cooled with water flowing vertically upwards, with inlet pressure 150 bar, inlet subcooling 70 K and mass flux 1000 kg m$^{-2}$ s$^{-1}$. Use saturated water properties at 150 bar pressure.

**Problem 1 (5 points):** Using the Bowring’s model for the onset of nucleate boiling and Jens&Lottes correlation for the subcooled boiling predict the locations of the Onset of Nucleate Boiling ($z_{\text{nb}}$) and the Fully Developed Boiling ($z_{\text{fdb}}$) in the channel.

**Problem 2 (5 points):** Using the homogeneous equilibrium model (HEM) calculate the total pressure drop in the channel.