10. Nuclear Reactor Core Dynamics

Almost all industrial processes in general and coolant flow in nuclear reactor cores in particular are subject to temporal changes and have an unsteady-state nature. Obviously transient or unsteady-state processes in nuclear cores are, except for very specific situations, undesirable and should be avoided. This is particularly true for those transients, which result from various types of instabilities. The purpose of this Chapter is to discuss and analyze two-phase flow transients and instabilities of two-phase systems.

10.1 Analysis of two-phase flow transients

The analysis of transients in boiling loops and systems involves the modeling of several coupled phenomena such as: single-phase and two-phase flows in heated channels and volumes, boiling heat transfer, phase separation and mixing, transient heat conduction in solid structures, effects of void fraction on power generation (e.g. due to neutronic feedback in BWRs), and others. In this section the basic equations that describe all these processes will be given.

The dynamics of a boiling channel is usually based on a one-dimensional drift-flux or homogeneous model. The following conservation equations are used in the one-dimensional drift-flux model

Mass conservation equation for liquid phase

\[ \frac{\partial}{\partial t} \left[ \rho_l (1 - \alpha) A \right] + \frac{\partial}{\partial z} (\rho_l j_l A) = -\Gamma A \]  

(10.1.1)

Mass conservation equation for gas phase

\[ \frac{\partial}{\partial t} \left[ \rho_g \alpha A \right] + \frac{\partial}{\partial z} (\rho_g j_g A) = \Gamma A \]  

(10.1.2)

Energy conservation equation for liquid phase

\[ \frac{\partial}{\partial t} \left[ \rho_l H_l - p (1 - \alpha) A \right] + \frac{\partial}{\partial z} (\rho_l H_l j_l A) = q_l^* P_H \]  

(10.1.3)

Energy conservation equation for gas phase

\[ \frac{\partial}{\partial t} \left[ \rho_g H_g - p \alpha A \right] + \frac{\partial}{\partial z} (\rho_g H_g j_g A) = q_g^* P_H \]  

(10.1.4)

Momentum conservation equation for two-phase mixture

\[ \frac{\partial}{\partial t} \left[ \rho_{lg} \frac{v_{lg}}{T_{lg}} A \right] + \frac{\partial}{\partial z} \left( \frac{\rho_{lg} v_{lg} j_{lg} A}{T_{lg}} \right) = \frac{\partial}{\partial z} \left( \frac{\rho_{lg} v_{lg}^2 A}{T_{lg}} \right) \]  

(10.1.5)
\[
\frac{\partial G}{\partial t} + \frac{1}{A} \frac{\partial}{\partial z} \left( G^2 A \right) + \frac{\partial p}{\partial z} + \left[ \phi_{\text{lo}}^2 \frac{4C_{f,\text{lo}}}{D} + \sum_{i=1}^{n} \xi_i \phi_{\text{lo},i}^2 \delta(z - z_i) \right] \frac{G^2}{2\rho_l} + \rho_m g \sin \varphi = 0
\]  
(10.1.5)

Closure relationships

\[ \alpha = \frac{j_g}{C_0 j + U_{ji}} \]  
(10.1.6)

where

\[ j_g = \frac{Gx}{\rho_g} \]  
(10.1.7)

\[ j_i = \frac{G(1-x)}{\rho_l} \]  
(10.1.8)

For homogeneous flow model the conservation equations are as follows

**Mass conservation equation**

\[ \frac{\partial \rho_m}{\partial t} + \frac{\partial G}{\partial z} = 0 \]  
(10.1.9)

**Energy conservation equation**

\[ \frac{\partial (\rho_m H_m)}{\partial t} + \frac{\partial (GH_m)}{\partial z} = - \frac{q^* P_m}{A} + \frac{\partial p}{\partial t} \]  
(10.1.10)

**Momentum conservation equation**

\[ \frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left( \frac{G^2}{\rho_m} \right) = - \frac{\partial p}{\partial z} - \left[ \phi_{\text{lo}}^2 \frac{4C_{f,\text{lo}}}{D} + \sum_{i=1}^{n} \xi_i \phi_{\text{lo},i}^2 \delta(z - z_i) \right] \frac{G^2}{2\rho_l} + \rho_m g \sin \varphi \]  
(10.1.11)

Closure relationship

\[ H_m = H_i(1-x) + H_g x \]  
(10.1.12)

**Exercise 10.1.1:** Find mixture density as a function of axial distance and time for a boiling channel with a step change in inlet velocity. Assume uniformly heated channel with saturated inlet conditions, homogeneous flow model and constant system pressure.
10.2 Analysis of two-phase system instabilities

Various thermal-hydraulic instabilities have been observed in two-phase flows. As already mentioned, instabilities are undesirable since they may degrade system control and performance, erode thermal margins and lead to mechanical damages. Nuclear reactors have to be designed and operated in such a way that sustainable instabilities are avoided. This requires a good understanding of the phenomena that govern various modes of instabilities in nuclear power plants as well as knowledge of stability margins, which are valid for particular system and operating conditions.

Classification of instabilities:

Two-phase instabilities can be conveniently classified into static instabilities, which can be explained in terms of steady-state laws, and dynamic instabilities, which require a consideration of the transient conservation equations.

Static instabilities

1. **Excursive (i.e., Ledinegg) instabilities** are non-periodic flow transients. Instabilities of this type plagued early low-pressure fossil boilers, since flow excursions could lead to burn-out of the boiler tubes. Ledinegg instability may occur in heated channels with low system pressure and low inlet loss coefficient, where pressure drop may decrease with increasing flow. If the pump characteristics has less negative slope, excursive instability will occur.

2. **Flow regime relaxation instabilities** are caused by the pressure-drop characteristics of the different flow regimes. For example, pressure drop in slug flows is less then in bubbly flow with the same flow rates of gas and liquid. If a system is operating in the bubbly flow regime near the flow regime boundary, a small negative perturbation in liquid flow rate may cause a transition to slug flow. As a result, pressure drop in the channel will reduce. If the channel operates at a constant pressure drop condition (as in case of a large number of parallel channels), more liquid will enter the channel to satisfy the boundary conditions. This, in turn, may cause the system to return to bubbly flow regime.

3. **Nucleation instabilities** include bumping and geysering phenomena. These instabilities are characterized by a periodic relaxation of the metastable condition that builds up due to insufficient nucleation sites. In particular, if the liquid superheat builds up until the existing nucleation sites are activated, rapid boiling and expulsion of the resultant two-phase mixture may occur.

Dynamic instabilities

1. **Density-wave oscillations** can occur in both diabatic and adiabatic two-phase systems and in diabatic single-phase systems. Generally speaking, density wave oscillations are caused by the lag introduced into the thermal-hydraulic system by the finite speed of propagation of density perturbations. This type of instability is one of the most important and of practical concern in BWRs and will be discussed in more detail in the following part of this section.
2. **Pressure-drop oscillations** can occur in loops having a negative slope (similar to the situation described for the excursive instability) and containing a compressible volume (e.g. an accumulator). In such systems excursions may occur periodically.

3. Flow regime excited instabilities can occur when a particular flow regime, normally slug flow, induces a periodic disturbance in the system operating state. If this disturbance is at a frequency that is close to the natural frequency of the two-phase system, a resonance can occur.

4. **Acoustic instabilities** may occur in two-phase system having the proper combination of geometric characteristics and sonic speed. As in single-phase gas flows, organ-pipe-type standing waves can be set up when a pressure pulse propagates through two-phase mixtures flowing in a conduit. On reaching an area change or obstruction, the change in acoustic impedance causes a pressure pulse of opposite polarity to propagate in the opposite direction. If the excitation frequency and geometry of the conduit is such that an integral number of one-quarter wavelengths can fit within it, the standing waves may appear. Such acoustic-induced channel pressure drop oscillations of large amplitude have been observed for subcooled systems operating in the negative-slope region of the system pressure-drop versus flow curve.

5. **Condensation-induced instabilities** are known to lead to large water-hammer-type loads, however, their nature is not fully understood. A typical example is the so-called chugging phenomena that has been observed in the vent pipes of steam relief valves which are submerged in a liquid pool. When the steam first exits into the subcooled pool of liquid it is normally at a high enough velocity to form a jet within the pool. However, as the steam flow rate drops off, the condensation rate in the pool may be large enough to completely collapse the steam jet, and cause a liquid slug to surge up into the discharge line. Subsequently, the steam can heat up the interface of the liquid slug to saturation, allowing the pressure of the discharging steam to increase such that it blows the slug back into the liquid pool. A cyclic process can occur with large inertial loads associated with the liquid slug motion being transmitted to the walls of the vessel containing the pool.

**Density-wave oscillations**

To understand the mechanisms of density wave oscillations, consider an air/water flow channel connected to a tank filled with water, (see Figure 10.2.1, depicting a system investigated by Svanholm and Friedly). Water is entering into the pipe through an inlet orifice and gas is constantly introduced into the pipe and mix with water at a certain distance downstream from the inlet orifice. To simplify the analysis, the following assumptions are made. The density of the air is negligible in relation to that of the water and the volume occupied by the water is negligible compared to that of the air. Finally, slip is also neglected.
The airflow determines the velocity of the two-phase mixture and the density of the mix is proportional to that of the liquid. Assuming that the pipe has no losses except at the inlet and exit. The hydraulic head loss at the inlet orifice is,

$$\Delta p_{in} = K_{in} \frac{U_{in}^2 \rho_i}{2}. \quad (10.2.1)$$

The pressure drop at the exit is,

$$\Delta p_{out} = K_{out} \frac{U_{out}^2 \rho_{out}}{2}. \quad (10.2.2)$$

Here $U_{in}$ and $U_{out}$ are mean inlet and outlet velocities, respectively, $K_{in}$ and $K_{out}$ are the inlet and outlet pressure loss coefficients, respectively, and $\rho_{out}$ is the outlet (two-phase) density. The constant pressure head, provided by the water tank, is always equal to the sum of these two pressure drops, i.e.

$$\Delta p_{tot} = \Delta p_{in} + \Delta p_{out} = \rho_l gh. \quad (10.2.3)$$

For example, opening the inlet orifice for a short time induces a density wave that propagates through the pipe and passes the exit orifice after a time $\tau$. During the passage, the corresponding pressure drop at the exit increases temporarily. Since the total pressure drop always remains constant, the increased pressure drop over the exit will bring a corresponding pressure decrease at the inlet. This means that less water is sprayed into the channel creating a sudden decrease in density, which induces the wave that propagates through the channel. This dynamic process causes density wave propagation in the channel.
If the amplitude of the waves is converging the system is said to be stable and if the amplitude is diverging it is said to be unstable. These finite propagation times, $\tau$, induce time-lag effects and phase-angle shifts between the channel pressure drop and the inlet flow, which may cause self-exciting oscillations.

In general, any increase in the frictional pressure drop in the liquid region has a stabilizing effect, since this pressure drop is in phase with the inlet flow and acts to damp the fluctuations. Inlet orificing can be used to stabilize an unstable flow. An increase in the two-phase region pressure drop has a destabilizing effect, since the pressure drop is out of phase with the inlet flow, due to the wave propagation time, $\tau$. Thus, an exit flow restriction is a strongly destabilizing factor.

For a fixed channel geometry, an increase in inlet velocity has a stabilizing effect in terms of heat flux, because the extent of the two-phase flow region and the density change due to boiling are significantly reduced by the increase in inlet velocity. An increase in the system pressure has a stabilizing effect in terms of exit quality, since at higher pressure the density change due to phase change is less significant.

Density-wave instabilities can be further classified as follows:

1. Loop instabilities
2. Parallel-channel instabilities
3. Channel-to-channel instabilities
4. Neutronically-coupled instabilities

The most important modes of density-wave instabilities are loop and parallel-channel instabilities. The parallel-channel mode corresponds to a system of a big number of channels connected in parallel, in which a constant pressure condition governs flow through each of the channels. The principles of density-wave instability that are described in this section correspond just to this mode of instability. The loop instability is very similar, however, the boundary condition of zero pressure drop in the loop is imposed.

**Methods of analysis of two-phase flow instabilities**

A rigorous stability analysis of boiling systems is difficult due to the nonlinear mathematical form of the underlying conservation equations and is only possible if some simplifying assumptions are made. In particular, if the threshold of instability is of interest, linearized models are often used. Such models are obtained by perturbing the governing equations around a given steady-state operating point. The linearized model is next Laplace-transformed and a frequency domain methodology is used to study the system stability.

While the linear stability analysis can be used to determine the instability threshold, this approach does not provide information concerning other characteristics of nonlinear
systems, such as the magnitude and frequency of any limit cycle oscillations. For this purpose a non-linear stability analysis must be performed.

In general, the methods of linear analysis of two-phase flow instabilities consists of the following steps (so-called frequency-domain methodology):

1. Linearize governing equations around steady-state operating point
2. Obtain transfer functions
3. Examine properties of roots of characteristic equations

The methods of nonlinear analysis of the two-phase instabilities are as follows:

1. Hopf’s bifurcation method
2. Method of Liapunow
3. Harmonic quasi-linearization (describing function methods)
4. Fractals as measure of strange attractors (chaotic vibrations)

Nonlinear methods are beyond the scope of the present course and will not be discussed (the interested reader may consult literature, e.g., “Boiling Heat Transfer – Modern Developments and Advances”, R.T. Lahey, Jr., Ed, Elsevier, 1992). In what follows some ideas behind the linear methods will be discussed.

**Frequency domain methodology**

Frequency domain methodology uses approaches that have been developed and should be known from the feedback-control theory. As a short reminder, a generic block diagram of a negative feedback-control system is shown in Fig. 10.2.2. This is a simple linear system with a single input and single output (SI/SO).

\[
X_{\text{out}}(s) = \left[ X_{\text{in}}(s) - H(s) \cdot X_{\text{out}}(s) \right] \cdot G(s),
\]

From which the output signal can be obtained as
\[
X_{\text{out}}(s) = \left[ \frac{G(s)}{1 + G(s) \cdot H(s)} \right] X_{\text{in}}(s). \tag{10.2.5}
\]

The transfer function \( G/(1+GH) \) is called the closed loop transfer function and \( 1+GH=0 \) is the characteristic equation of the close loop. If any of the roots of \( 1+GH \) will appear in the right-halve of the complex plane, the closed system will be unstable.

**Perturbation analysis**

The first order perturbation of a time dependent function \( f(t) \) can be viewed as a first order Taylor’s series expansion,

\[
\delta f \equiv f(x(t)) - f_0 = \left. \frac{\partial f}{\partial x} \right|_{x_0} \delta x(t), \tag{10.2.6}
\]

where \( f_0 \) is the value of the unperturbed function. Consider a few examples:

Let \( f(x) = c_1 x^c \). The perturbed equation is,

\[
\delta f(x) = ac_1 x_0^{c-1} \delta x \iff \delta f = a \frac{\delta x}{x_0}.
\]

Let \( f(x) = \frac{dx}{dt} \); then,

\[
\delta \left( \frac{dx}{dt} \right) = \frac{d(\delta x)}{dt}. \tag{10.2.7}
\]

Let \( f(x) = \int_{g_1(x)}^{g_2(x)} G(x,z) \, dz \); then,

\[
\delta f(x) = \int_{g_0}^{g_2} \delta G(x,z) \, dz + G_0(x,g_2(x)) \delta g_2 - G_0(x,g_1(x)) \delta g_1. \tag{10.2.8}
\]

Perturbations are quite similar to differential calculus. The only real difference is that perturbations are taken around steady state point and all variables not perturbed are denoted by a subscript zero to indicate the steady state.

**Derivation of perturbed equations**

The time-dependent pressure drop in a boiling channel can be obtained from the integration of the momentum equation (e.g. Eq. (10.1.5) or (10.1.11)). The integration can be simplified by dividing the channel into two parts: a single-phase flow part, which is stretching from the inlet to the boiling boundary, located at distance \( \lambda \) from the inlet,
and the two-phase flow part, stretching from the boiling boundary to the outlet from the channel, see Fig. 10.2.3.

![Diagram showing single-phase and two-phase sections of a boiling channel.]

Figure 10.2.3. Single-phase and two-phase sections of a boiling channel.

Using the homogeneous model, the pressure drop across each section is given as,

$$\left(\Delta p_H\right)_i = p_{in} - p(z) = \int_0^L \left\{ \frac{\partial G}{\partial t} + \frac{1}{\rho_l} \frac{\partial G^2}{\partial z} + \frac{4C_{f,jo}}{D_h} \frac{G^2}{2\rho_l} + \rho_m g \right\} dz + \sum_{i=1}^{N_{th}} \frac{1}{2\rho_i} G^2(z_i), \quad (10.2.9)$$

for the single phase flow part and

$$\left(\Delta p_{2ph}\right)_i = p(z) - p_{ex} = \int_0^L \left\{ \frac{\partial G}{\partial t} + \frac{\partial}{\partial z} \left( \frac{G^2}{\rho_m} \right) + \frac{4C_{f,jo}}{D_h} \frac{G^2}{2\rho_m} + \rho_m g \right\} dz + \sum_{i=1}^{N_{th}} \frac{1}{2<\rho_m(z_i)>} G^2(z_i), \quad (10.2.10)$$
for the two-phase flow part. Here \( \lambda \) is the position of the boiling boundary. Combining Equations (10.2.9) and (10.2.10) with equations for mass and energy conservation, perturbing around a steady state point and then Laplace transforming yields the following general expressions for the single-phase and two-phase pressure drop perturbations,

\[
\delta (\Delta P_{1\theta})_{H} = \Gamma_{1,H}(s) \delta j_{in} + \Gamma_{2,H}(s) \delta \tilde{q}_{H}^{\prime\prime} + \Gamma_{3,H}(s) \delta \tilde{h}_{in},
\]

\[
(10.2.11)
\]

\[
\delta (\Delta P_{2\theta})_{H} = \Pi_{1,H}(s) \delta j_{in} + \Pi_{2,H}(s) \delta \tilde{q}_{H}^{\prime\prime} + \Pi_{3,H}(s) \delta \tilde{h}_{in}.
\]

\[
(10.2.12)
\]

Here \( \Gamma_{1,H}, \Gamma_{2,H} \) and \( \Gamma_{3,H} \) are transfer functions of the perturbations of inlet velocity, heat flux and inlet enthalpy, respectively, on the perturbation of the single-phase pressure drop. \( \Pi_{1,H}, \Pi_{2,H} \) and \( \Pi_{3,H} \) are the corresponding transfer functions for the perturbation of the two-phase pressure drop. Derivation of Eqs. (10.2.11) and (10.2.12), as well as expressions for transfer functions can be found in “Boiling Heat Transfer – Modern Developments and Advances”, R.T. Lahey, Jr., Ed, Elsevier, 1992.

**Stability criterion**

As an example, parallel channels with constant power and constant inlet enthalpy will be considered. In such case, \( \delta \tilde{q}_{H}^{\prime\prime} = \delta \tilde{h}_{in} = 0 \) and since pressure drop is constant,

\[
\delta (\Delta P_{1\theta})_{H} + \delta (\Delta P_{2\theta})_{H} = 0.
\]

\[
(10.2.13)
\]

Combining Eqs. (10.2.11) and (10.2.12) with (10.2.13) yields

\[
[\Gamma_{1,H}(s) + \Pi_{1,H}(s)] \delta j_{in} = 0.
\]

\[
(10.2.14)
\]

In the case of parallel channel instabilities density-wave oscillations are manifested by self-sustained oscillations in the channel inlet flow rate caused by a feedback between the pressure drop perturbations in the single and two-phase portions of the channel. Using the technique of linear system control theory, the parallel channel stability can be considered as a feedback system. The appropriate block diagram is shown in Fig. 10.2.4.

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**Figure 10.2.4.** Block diagram for the parallel channel model.

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Page 10-10
The block diagram shown in this figure yields the following relationship between the external perturbation $\delta j_{\text{ext}}$ and the system response, $\delta j_{\text{int}}$,

$$
\delta j_{\text{int}} = \left[ \frac{1}{\Pi_{1,H}(s)} \right] \delta j_{\text{ext}},
$$

(10.2.15)

It follows from this equation that the characteristic equation of the boiling channel is,

$$
1 + \frac{\Pi_{1,H}(s)}{\Gamma_{1,H}(s)} = 0.
$$

(10.2.16)

The necessary and sufficient condition for the boiling system under consideration to be stable to small perturbations is that all the roots of the characteristic equation, Eq. (10.2.16) have negative real parts.

In the case of complicated transcendental algebraic equations, such as those that occur in Eq. (10.2.16), direct evaluation of roots is not trivial, and approach using Nyquist Criterion has proven to be an efficient way of investigating system stability.

Ignoring some mathematical details, the Nyquist Criterion can be expressed as follows:

Let $G(s)$ be an analytical function on a closed contour, $C$, where $C$ consists of the imaginary axis, $s = j\omega$, $(-\infty < \omega < \infty)$, and a large semicircle $S = R \cdot e^{j\omega}, (-\pi/2 \leq \varphi \leq \pi/2; R \to \infty)$. Let also assume that $G(s)$ is analytical within $C$, except for, at most, a finite number of poles. Then, the number of clockwise encirclements ($N$) of the origin by the mapping $G(s)$, per a single traverse along the contour $C$ in the $s$-plane, is equal to the difference between the number of zeros ($Z$) and that of poles ($P$) of $G(s)$ within $C$.

For boiling parallel channel,

$$
G(s) = 1 + \frac{\Pi_{1,H}(s)}{\Gamma_{1,H}(s)},
$$

(10.2.17)

and its poles are equivalent to the combined poles of $[\Pi_{1,H}(s)+\Gamma_{1,H}(s)]$ and zeros of $\Gamma_{1,H}(s)$. From the shape of transfer functions $\Pi_{1,H}(s)$ and $\Gamma_{1,H}(s)$ is clear that all the singularities of $[\Pi_{1,H}(s)+\Gamma_{1,H}(s)]$ are removable, whereas $\Gamma_{1,H}(s)$ has no zeros within $C$, thus, $P = 0$. Consequently, the parallel channel model derived herein is stable if, and only
if, the Nyquist plot of $\frac{\Pi_{1,H}(s)}{\Gamma_{1,H}(s)}$ does not encircle the point (-1,0). Figure 10.2.5 shows examples of Nyquist plots for stable and unstable channels.

![Nyquist plot diagram](image)

**Figure 10.2.5. Nyquist plots of $\frac{\Pi_{1,H}(s)}{\Gamma_{1,H}(s)}$ for stable and unstable channel.**

**Time domain methodology**

In addition to the frequency-domain approach, the time-domain methodology is used in practical calculations of the nuclear reactor stability. In such calculations, the dynamic behavior of the system caused by a certain perturbation of input parameters is calculated as a function of time. Fig. 10.2.6 shows an example of the perturbation of an input parameter (this could be for instance the inlet subcooling) and an example of the output parameter (this could be for instance the reactor power).
Figure 10.2.6 Examples of an perturbation and resulting answer.

The answer shown in Fig. 10.2.6 is typically obtained from an analysis performed with a transient code where both reactor kinetics and reactor thermal-hydraulics equations are solved simultaneously. If the answer has an oscillatory character then after a certain period of time the least damped eigenfrequency will dominate. The amplitude ratio, or the so-called decay ratio, \(A_2/A_1\) is a practical measure how effective the damping is and how far from the instability the system is. For undamped system \(A_2/A_1 = 1\). In such case the system is at the threshold of the instability. For decay ratio \(A_2/A_1 < 0.25\) the system is well damped.

**Stability map of a boiling channel**

It is instructive to represent the dynamic behavior of a boiling channel on a so-called stability map. One such map proposed by Ishii and Zuber is shown in Fig. 10.2.7. The map shows the region where the boiling channel is unstable in function of two non-dimensional parameters: the subcooling number and the phase-change number. The subcooling number is defined as follows,

\[
N_{\text{sub}} = \frac{(\rho_l - \rho_g)(H_f - H_0)}{\rho_g H_{fg}},
\]

(10.2.18)

and the phase-change number as,
Here $L_H$ is the heated length, $P_H$ is the heated perimeter, $q_0^*$ is the heat flux, $A$ is the channel cross-section area, $j_{in}$ is the inlet velocity, $H_0$ is the inlet enthalpy and $H_{lg}$ is the latent heat.

It can be seen in Fig 10.2.7 that stability boundary curve for higher inlet subcoolings is nearly parallel with the line of a constant exit quality $x_{ex}$, given by,

$$N_{sub} = N_{pch} - \frac{\rho_l - \rho_g}{\rho_g} x_{ex}. \quad (10.2.20)$$

Ishii used this observation and derived a simple stability criterion for the high inlet subcoolings as follows,

$$x_{ex} \leq \left[ \frac{2(\xi_{in} + 2\varphi + \xi_{out})}{1 + \varphi + \xi_{out}} \right] \frac{\rho_g}{\rho_l - \rho_g}. \quad (10.2.21)$$

The boiling channel is stable as long as inequality (10.2.21) is satisfied. Here $\xi_{in}$ and $\xi_{out}$ are the inlet and outlet loss coefficients, respectively and $\varphi$ is the friction number given as,
\( \varphi = \frac{C_f \, L_H}{2 \, D} \) \hspace{1cm} (10.2.22)

**Exercise 10.2.1:**
Plot the stability map for a boiling channel with length \( L_H = 3.6 \) m, hydraulic diameter \( D = 10 \) mm and pressure \( p = 70 \) bar. The inlet loss coefficient is equal to 4 and the outlet is equal to 0. Use the Blasius formula for the Fanning friction factor \( C_f \) assuming \( \text{Re} = 10^5 \). What is the maximum exit quality to keep the channel stable?